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The Franklin Institute

1826

1955

Some Prominent Members of The Franklin Institute 2. Joseph Saxton, 1799–1873
THOMAS COULSON

A Postulational Approach to Electromagnetism
PARRY MOON AND DOMINA EBERLE SPENCER

Detailed Example of Interconnecting Piece-Wise Solutions
GABRIEL KRON

Beams of Metal or Wood Subjected to Millions of Reversals of Compression and Tension ($\sigma_c = -\sigma_t$) or to Longtime Steady Compression Longitudinally
ARTHUR BUCHWALD LAMBERG

Franklin Institute:
Research Notes
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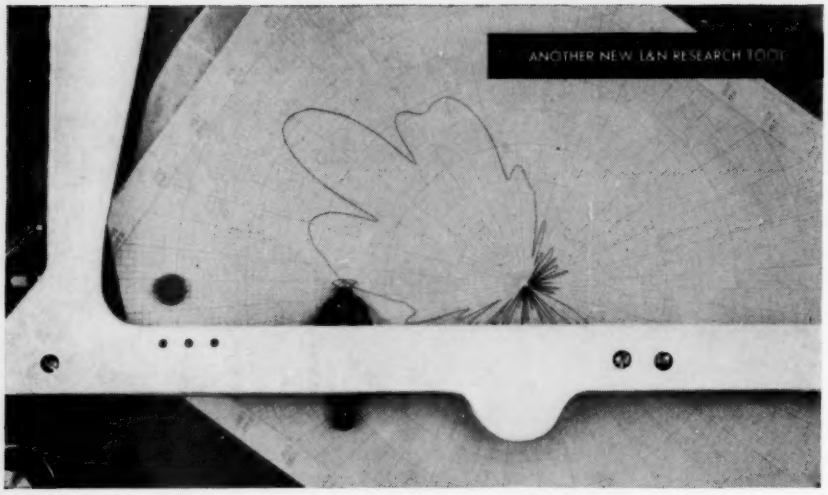
VOL. 259 NO. 4



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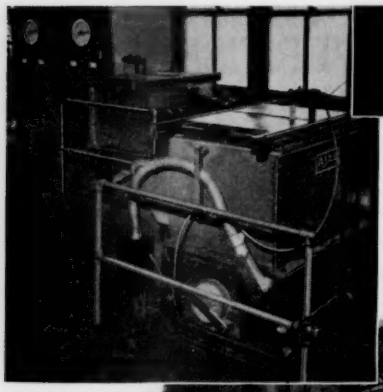

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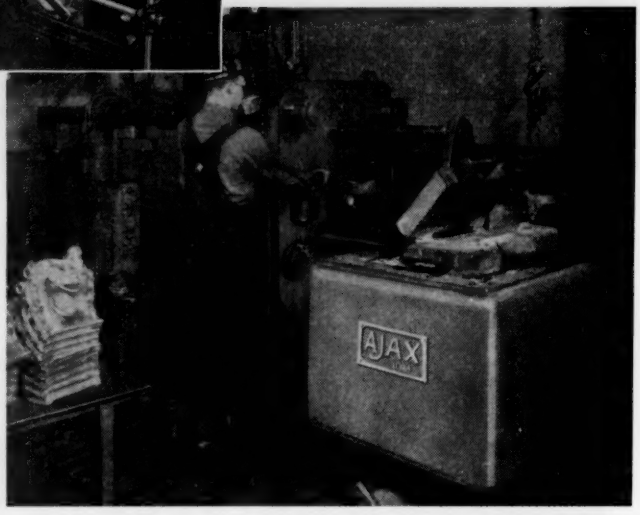

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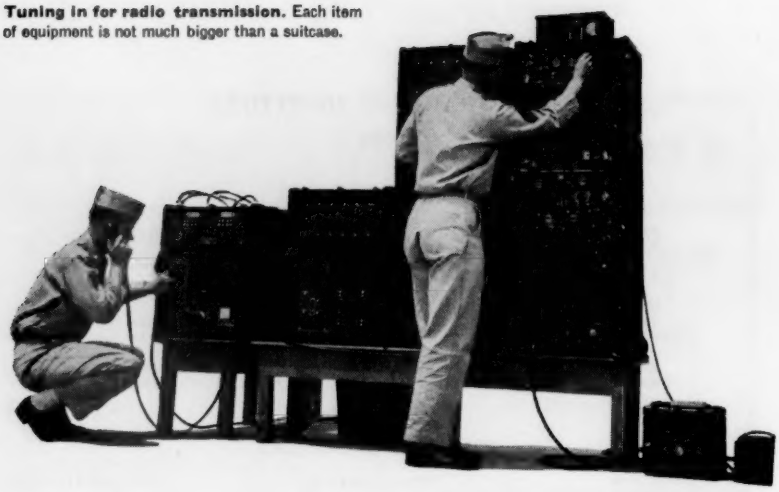

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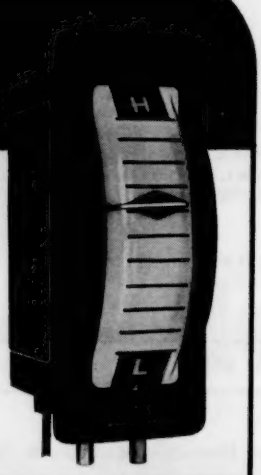

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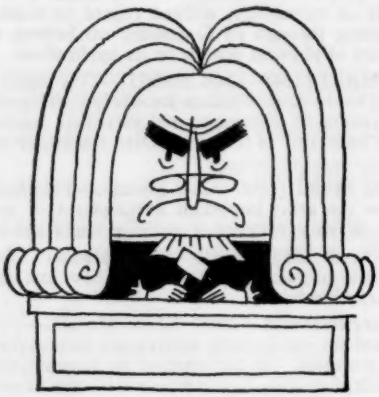

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Vol. 259 APRIL, 1955 No. 4

SOME PROMINENT MEMBERS OF THE FRANKLIN INSTITUTE
2. JOSEPH SAXTON, 1799–1873

BY
THOMAS COULSON[1]

Born at Huntingdon, Pennsylvania, then a small village on the bank of the Juniata River, on March 22, 1799, Joseph Saxton was the second son in the eleven children born to an English father and a mother (Hannah Ashbough) of German extraction. He died in Washington, D. C. in 1873. During the three-quarters of a century spanned by his life, the world made greater progress in the applications of science to industry than in any similar period of its previous existence. It was a time of great beginnings, when the steamboat, the railroad, the electrical telegraph, the photograph, the spectroscope were developed, and when interchangable parts were introduced into manufacturing processes.

Like other men of his acquaintance who gained fame in science, as Michael Faraday and Joseph Henry, he had few advantages in education—nothing more than an ordinary village school had to offer. This did not advance beyond the stage of rudimentary reading, writing, and arithmetic. His subsequent achievements proved what can be done on this slight foundation by a man who has the natural ability to grapple with ideas and the imagination to perceive how they can be extended.

Some of his talent might have been inherited, for his father, James Saxton, seems to have been a man of parts with an adaptable nature that was expended in too many directions to attain a conspicuous success in any one. Among other occupations he held an inferior banking post, was a justice of the peace, and eventually became the proprietor of the first nail factory to be erected in the vicinity of Huntingdon. Nails, it should be remembered, had been hand-made until just before this and, because of their scarcity, were used sparingly in building construc-

[1] Director of Museum Research, The Franklin Institute, Philadelphia, Pa.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

tion. The Saxton nail factory was something of a pioneer enterprise that had to compete with the imported product of sweated Sheffield labor.

Joseph left school at the age of twelve and was given employment in his father's factory. Tradition has it that the boy perceived some defects in the primitive mechanisms employed, and suggested how they might be remedied. When he had convinced his father that the recommendations, if applied, would increase the efficiency of the plant, they were put into practice with satisfactory results.

Youth was as restless in those days as it is today, and just as rebellious against monotonous routine. Joseph protested against the drudgery of nail-making, which offered no prospect of variety nor any call for skill. He pleaded to be enabled to learn a trade where he could exercise the talent for mechanics he had already displayed. In the outcome he was apprenticed to the local watchmaker. Two years in this congenial occupation gave him an aptitude in the handling of small parts and improved his knowledge of mechanical movements. Then his employer died. As no other watchmaker appeared in the village to take his place, Joseph was thrown out of employment.

While waiting for another job he interested himself in the construction of a printing press. When this was completed, the young Saxton conceived the idea of issuing a newspaper. This was not a success. He may have had the physical equipment of a printer and the literary ability of an editor, but he was lacking in the qualities of a publisher. The newspaper failed to gain a wide distribution because the publication dates were eccentric. Probably it only appeared on those days when the weather was too inclement to permit the owner to follow his favorite outdoor pursuits.

The woods, the river, and the fields were irresistible attractions to the boy. He acquired a devotion to outdoor sports that lasted as long as his health. In his boyhood days the love of hunting was the cause of an accident that might easily have proved fatal, but which, happily, served to teach him that he possessed the power of original thought.

As a reward for his industry, his father had given him a new rifle. In loading it for the first time he had made the grease patch for the bullet with such care that it would not allow air to escape in the barrel. As a consequence, each time he rammed the bullet it would spring back from the expansion of the compressed air. At length, the ramrod was thrown clear of the barrel after an unusually violent thrust. Determined to force down the bullet against the resistance, he set the projecting end of the ramrod against the trunk of a tree and threw all his weight against the gunstock. The result was entirely unexpected.

By some stroke of good fortune he escaped serious injury from the explosion that followed this act. After picking himself up and gathering together his wits, he began to reflect upon the possible causes of the

mishap. He drew an analogy from his experience in hammering nails. He knew that the force applied to the nail caused a rise in temperature, and came to the conclusion that the violent pressure he had employed against the bullet had heated the column of air behind it to a point where it caused the gunpowder to ignite.

When Saxton was subsequently assured by a competent person that his line of reasoning was correct, he awakened to the knowledge that he possessed the power of original thought. He was so impressed by this discovery that he resolved to put it to use in his occupation.

The trouble was that, back in the nail factory, he had no occupation in which original thought might be applied with profit, nor did the surrounding neighborhood offer brighter prospect. He had reached the age of eighteen, manhood in those days. If he were to find an outlet for his newly discovered talent, he must seek it in a broader field. The nearest important center of labor was Philadelphia, more than 150 miles away. He resolved to make his way there in spite of a shortage of funds to pay for the journey and to maintain himself while seeking employment. However, to a young man of resolute will the handicaps were not insurmountable. A few years earlier he had built himself a boat on the model of a ship-of-war. This was repaired, re-modelled, and provisioned. Against parental advice, he resolved upon making the attempt to reach Philadelphia by water. Moreover, he persuaded two of his friends to join him in the adventure.

The journey was uneventful until the wayfarers reached Harrisburg where, to their dismay, they found the river was too turbulent to permit any thought of further navigation. Their provisions were running short, too. Saxton later used to tell with a certain amount of suppressed mirth how they escaped from their dilemma. In order to obtain the necessary funds to continue the journey the youths offered to sell the boat to a German innkeeper for ten dollars and free board for the night. He was willing to buy at the price but demanded to be shown the boat. Not experienced in nautical matters, he stepped on the gunwale to climb aboard, thereby turning the boat over and plunging him into the river. When he was rescued by the young men, he would have nothing to do with such a capricious thing as an unstable boat. Dry clothes, wifely ministrations, and the traditional alcoholic remedies against catching a cold, put him in better humor, and the transaction was completed. The trio of adventurers finished their journey by more orthodox ways.

What became of his two companions is not known, but young Saxton soon began to make a name for himself. He soon found employment. He carried a letter of introduction to a Chestnut Street watchmaker who was so impressed by the warmth of the recommendation it gave Saxton that he set the young man to work immediately. It would appear that the enterprising youth had been doing some seri-

ous thinking on mechanical subjects, for he very soon produced his first invention. This was a machine for cutting the teeth on wheels the outlines of which were true epicycloidal curves. Intended for the construction of chronometer wheels, it had other applications. Although this machine cannot now be identified it was highly praised by Joseph Henry for the profound thought and remarkable ingenuity displayed in its design.

For some reason or other that has not been explained, Saxton now decided to change his employment, and went to work for an engraver where he could exercise a natural talent for drawing. At odd intervals during his life he amused himself by sketching scenery. However, he did not remain long in this occupation. He was soon back in more congenial employment with one of the most famous of Philadelphia clockmakers, Isaiah Lukens. While with this employer he constructed an astronomical clock equipped with a compensating pendulum and an escapement of his own design. He is said, also, to have had a hand in the design of the clock in Independence Hall.

Saxton was not a man of gregarious habits but he was always ready to seek the companionship of men who were wiser and more experienced than himself. It was natural that he should have been attracted to the society offered by membership in The Franklin Institute. As he was already known for his ingenuity he was welcomed into the friendship of Alexander Dallas Bache, J. Vaughan Merrick, Frederick Fraley, and Professor Cresson, who were the leaders of Institute activities and prominent members of the American Philosophical Society.

Although there was a freshening of interest in technology in the United States, the Americans were still looking to Europe for leadership in science. Saxton concluded that if his knowledge was to be advanced to a higher level he should visit England. By practicing the most rigid economies he was able to hoard enough money to pay his passage and to provide for a short residence in Europe. His intention was to devote his time to study.

However, banks operated then more than now under laws not to be apprehended by mathematicians and mechanics. No sooner had Saxton deposited his savings in a bank than it closed its doors, leaving him destitute and friendless in a strange land. Like the explosion of his rifle, this catastrophe set him upon a new course.

He quickly found employment at the Adelaide Gallery of Practical Science.

This was a period when the world was just as curious about new discoveries in the technical field as it is today. Interest was acute in the newer discoveries that gave promise of bringing fundamental changes into the everyday life of the people. It will be recalled that the period was unusually rich in discovery, especially in electricity. Those early years of the nineteenth century are sprinkled with great

names—Arago, Ampere, Davy, Oersted, Faraday and Ohm, among others. So spectacular were these revelations of electricity that they captured the imagination of the lay public.

There were, however, few channels of communication through which the information concerning these discoveries could reach the general public. There were no electric telegraphs, no radio, and few newspapers of great circulation. Efforts were made by some learned societies, notably the Royal Institution, to present demonstrations for the enlightenment of the members, but these were inadequate to gratify public curiosity. Private enterprise tried to fill the void. The Adelaide Gallery was founded in 1831 for the purpose of introducing the new phases of science to non-technical visitors who were willing to pay for the privilege of seeing its exhibits. The Gallery displayed a collection of "philosophical apparatus" designed to demonstrate the technical aspect of their subjects, but it also contributed a number of exhibits intended to amuse as well as to instruct. The most famous of these were the Persian rope dancer and Maillardet's oriental magician, a pair of automata that never failed to arouse the admiration of the young and the curiosity of their elders who were led to examine the ingenuity of these elaborate mechanisms.[2]

When Saxton began to work at the Adelaide Gallery he was employed upon the construction of an exhibit that would amuse the visitors, but which was a complicated piece of mechanism. This was called the "Paradoxical Head." The head and bust of a Turk had been carved in wood with some artistic skill. Around his otherwise bare neck the Turk wore a black ribbon with what appeared to be a seam running through the center. To the uninitiated the head and bust were carved from a single piece of wood, an illusion that was supported by the performance of a supposedly expert swordsman who, after a few introductory flourishes of his thin sword blade, passed this through the Turk's neck, apparently severing the head from the body. Yet the head was not detached. The public were left to solve the problem in their own way.

The "Paradoxical Head" aroused great wonder when it was placed upon display. Numerous conjectures were advanced in explanation of the trick, and the most commonly expressed opinion was that the head was held in place by magnetism, Mr. Sturgeon's electro-magnet being then in the public eye. However, the explanation resided in an ingenious arrangement of catches that Saxton had contrived. Instead of the swordsman lopping off the head with a clean cut, he deftly inserted the thin sword blade in the division between the Turk's head and body, which looked like a seam in the black neck ribbon, and with a pushing

[2] That the interest in these automata was not without justification will be appreciated by an examination of Maillardet's equally famous writing lady displayed in the Institute Museum.

stroke he unlocked and relocked a succession of catches in its passage, leaving the same connections as before the passage of the blade.

The success of the "Paradoxical Head" was sufficient to secure Saxton's retention for the construction of other exhibits for the Adelaide Gallery. Those which were evidently intended for the amusement of the visitors included a nautical scene in which a group of model ships apparently sailed upon a model sea. They were controlled by a concealed clockwork mechanism under the water. He also contrived a diving bell with glass sides in which visitors were lowered into a deep tank.

Not all his work was confined to amusing tricks. He contributed to the "philosophical apparatus." Moll in Holland and Henry in the United States had announced their improvements in the electro-magnet, and there was a demand upon the part of the general public to witness a performance of this scientific wonder. Saxton made a compound steel magnet which sustained a weight of 525 pounds, which placed it among the most powerful magnets in Europe at the time. He also constructed a magnetic needle several feet in length with a mirror at one end, which exhibited for the first time on a grand scale, by the movement of a reflected beam of light, the hourly variations of the earth's magnetic force.

The attendance at the Adelaide Gallery was not restricted to the lay public seeking to gratify their curiosity. Prominent engineers and mechanicians like Brunel, Telford, Whitwell, and Hawkins were also among the visitors, and some of these were sufficiently interested in Saxton's ingenious devices to seek him out and to make his acquaintance. With some of these men, notably the four whose names are given above, he formed intimate friendships. Through their influence he was introduced to the Royal Institution and its presiding genius, Michael Faraday.

By keeping his eyes and ears open he contrived to add materially to his knowledge of men, affairs, and science. His mind teemed with suggestions for new instruments and apparatus. In his letters home he mentioned many projects he lacked only time to undertake. Unlike so many contemporary men of science, Saxton did not isolate himself from the main currents of thought outside of his special field. The England in which he found himself was passing through the political turmoil that ended in the passage of the Reform Bill, and he took a lively interest in the discussions concerning the passage of the measure through Parliament. But residence in a foreign country did not make him oblivious to what was happening in his native land. He was always greedy for news from the United States.

While this political unrest was seething around him, he nevertheless found time to develop one of his greatest achievements, the magneto-electric machine.

Let us bear in mind the fact that Faraday had announced the discovery of magneto-electricity only in November, 1831. The presence of electricity produced by magnets had been detected by means of a galvanometer. In this experiment the impulse of induction existed only for the fraction of a second on the making and breaking of the circuit.

Saxton set himself the problem of creating a current of maximum intensity by making and breaking the circuit under proper conditions. The remarkable feature of his work was that he went directly to the solution of the problem apparently without any hesitation.

For the inducing force he used a laminated magnet of several strips of iron bent in the shape of a horseshoe and securely fastened together. For the armature, he bent a soft iron rod into the form of a U and wound each arm with 30 or 40 yards of insulated copper wire. The armature was caused to rotate by means of a simple crank driving-wheel and pulley.

A later experiment of Faraday's had demonstrated that when this armature was brought into contact with the poles of the magnet a transient electric current would be induced and, when the armature was withdrawn from the contact a current in the opposite direction would result. Saxton found that a higher voltage could be induced if the armature was made to rotate between the poles of the permanent magnet. Cutting across the lines of magnetic force at the poles, the rotating armature increased the induced voltage.

The little machine he developed produced 3.5 volts at 100 revolutions per minute. He exhibited it at the meeting of the British Association for the Advancement of Science held at Cambridge in 1833, where he demonstrated it to produce an unusually vivid spark, to decompose water, to make a good arc light, and to cause the more curious members to writhe under the effects of physical shocks that, in the absence of meters, was the only means of measuring the voltage. Samuel Taylor Coleridge, the poet and essayist, attended the meeting of scientists and, after speaking enthusiastically upon the wonders of electromagnetism, added words of high praise for Saxton's machine.

In August of the same year the machine was placed on exhibition at the Adelaide Gallery. Faraday, Daniell, Wheatstone, and other distinguished scientists spoke appreciatively of the machine. And well they might, for this modest little machine was the forerunner of the gigantic electrical generators of today.

Unfortunately, Saxton did not publish any account of his machine at the time he developed it. This neglect allowed a London instrument maker named Clarke to contrive a few minor re-arrangements of parts, and to publish a description of *his* machine in the *Philosophical Magazine* for August, 1836. By this action he received credit for the invention on the grounds of prior publication. Saxton did not surrender his claims

without resistance. He made a reclamation in the October issue of the same journal in which, after describing his own instrument and invoking the testimony of eminent men to support his claims to invention, he said:

> In conclusion, I think it will be evident from the preceding statement, that the magneto-electric machine, which Mr. Clarke has brought forward 'after much anxious thought, labor, and expense' is a piracy of mine; the piracy consisting not in manufacturing the instrument—for everyone is at full liberty to do so—but in calling it an invention of his own, and suppressing all mention of my name as connected with it.

However, the injury was done. The name of Clarke continued to be associated with the machine. Even Becquerel named him as the inventor when writing his history of electricity.

At the meeting of the British Association just mentioned, John H. Hawkins described a "locomotive differential pulley" invented by Saxton. The fundamental principle of this device can best be understood by supposing motion to be given by a cord to a small pulley, say six inches in diameter, around the axis of a wheel, say six feet in diameter. It is evident that if motion be given to the smaller pulley by unwinding the cord, the larger wheel will move with a velocity twelve times that of the smaller; in other words with twelve times the velocity of the impelling cord, and would give an equivalent velocity to another cord to which vehicles might be attached.

Hawkins offered some calculations from which he concluded that a continuous motion of thirty miles an hour could be given to a passenger vehicle by means of relays of horses, walking at the rate of $2\frac{1}{2}$ m.p.h. This seemed such an ideal way of providing speed for the appallingly slow travel of that generation that a company was promptly formed for the application of the principle to canal boats. However, the time for such an application was past. The Rainhill locomotive trials had been held and George Stephenson had demonstrated that a practical steam locomotive had been built. The speedy railroad age had opened, and the public was no longer interested in leisurely canal travel.

Saxton was now in charge of the apparatus at the Adelaide Gallery, and he began to produce a succession of observations and devices that are principally interesting because of the novelty and variety of his thought. He invented a dynamometer for measuring the speed of ships. He put the Perkins steam gun (which was exhibited at the Gallery) to a practical test, and found that when a stream of bullets was projected against an iron target, each bullet emitted a spark of light upon impact. He repeated this experiment in the dark, the better to observe the phenomenon. Inspired with a desire to produce a better weapon, he experimented with a device in which condensed carbonic acid gas was employed as the propellant. This was rejected as inefficient because he learned that the expansive force of the gas exhausted so much of its

energy in putting itself into motion that little force was left for propelling a projectile.

He kept a watchful eye on the ingenious Mr. Perkins. When the latter invented an apparatus for demonstrating the compressibility of water, Saxton arranged his own experiment with the same object in view. His apparatus consisted of a strong cast iron bottle into the neck of which a cylindrical plunger was fitted. The plunger was forced into the water-filled bottle by means of a hydrostatic press. The results were said to have been more accurate than those obtained by Oersted in Copenhagen.

Saxton had doubts about this experiment, suspecting that some water was forced into the pores of the vessel. He thereupon devised another experiment in which he filled entirely with water a small brass sphere with thick sides, and closed with a screw stopper. Having subjected the sphere to a sub-zero temperature during an entire night, he found in the morning that it was encased in a filament of ice. He believed that this proved the expansion of the water by freezing had forced some of the liquid through the pores of the container.

Allusion has been made to Saxton's youthful enthusiasm for outdoor sports. This interest had not waned with the passing of the years, and he found amusement in devising methods by which he could indulge in his pastimes. He contrived an instrument for projecting his fishing line into the river beyond the range of a normal cast. He cut a thin board, like a shingle, in the outline of a fish, and loaded it on one edge so that it would float vertically. From the lower edge he suspended baited hooks, and to the sides of his wooden fish he attached three short cords united to a long one, after the manner of fastening a kite. When the instrument was thrown into the stream, the fisherman ran along the bank holding an end of the long cord, and the artificial fish made off into the middle of the stream where it remained, for the same reason, he said, that a kite will remain in the air.

He also retained his interest in hunting, but his early experience with gunpowder had given him a respect for its properties, and he set himself the task of finding a less temperamental propellant. He invented an airgun small enough to slip into a hollowed-out walking cane, with which he developed a surprising rapidity and accuracy of fire.

He did not abandon the use of gunpowder for hunting, but he had ideas on economizing in its use. He pursued these in the direction of securing greater accuracy of fire. He realized this by producing a telescopic sight with hair lines for sighting.

It is generally supposed that the first metallic cartridge was Colonel Boxer's tin coiled brass case, invented about 1865. However, it is well known that Saxton was using a metallic cartridge case in connection with the rifle he had fitted with the telscopic sight, during his residence in London. Saxton resembled Franklin in declining to accept recom-

pense for his inventions. Had he patented his metallic cartridge he would have gained a substantial fortune from its adoption.

He seemed to dart off into unexpected directions. Among his other achievements while in London was the development of an apparatus for determining the height of water in a steam boiler. He placed a small hollow floating magnet in a copper tube, communicating at one end with the top and at the other end with the bottom of the boiler. Surrounding this tube was another of glass containing water and a second floating magnet, the position of which indicated the height of the water in the boiler. This idea was later patented by some one else and put into use.

Saxton also invented a machine for engraving from medals. He may have had in mind an improvement to the machine invented in 1817 by Gobrecht of the United States Mint, which could be used only when the medals were in low relief. Saxton's machine was more adaptable and could be applied when the medals were in either high or low relief. At a later date, this machine was made for operation by steam or water power. The engravings for the well known "Manual of the Gold and Silver Coins of all Nations struck within the last century" by Eckfeldt and Du Bois were all made with the aid of this apparatus.

In November, 1834, The Franklin Institute awarded him the Scott Legacy medal for the invention of a reflecting pyrometer. The following extract from the Committee's report describes the device and its operation.

> This instrument shows and measures, in a peculiar and advantageous manner, the linear expansion of a metallic or other rod subjected to the influence of heat. The rod resting against a fixed support at one end, the other end presses against a sliding bar, which carries an arm attached to one end of a fusee chain of a watch. This chain is wound around an axle carrying a mirror; and the other end of the chain is fastened to a spring to preserve its tension. Hence, as the rod under trial expands, and the sliding bar moves, the axle and mirror revolve, and if a sunbeam, thrown upon the mirror in the proper position, be reflected from it against a distant wall, the angular motion of the reflected image will be twice that of the axle, and will serve to measure the amount of expansion. As the sun is always in motion, a fixed mirror is made to reflect another beam, at first coinciding with the former one; and as the latter beam moves only with the sun, the angular distance between the two reflected beams or images will be twice the angular motion of the axle.
>
> This instrument is especially valuable for the trial of compensating pendulums as has been proved by Mr. Saxton. For this purpose the pendulum was enclosed in a hollow cylinder, in order that hot or cold water might be used for varying the temperature, the cylinder was supported vertically in a wooden frame; the lower end of the pendulum passing through a cork tightly closing the lower end of the cylinder, was adjusted to the sliding bar beneath it, which pressed firmly upward against the pendulum by the action of a spring. By this arrangement the revolving mirror was found always to return to its first position when

slightly moved by the hand; thus showing the delicacy of the mechanism, and the pendulum was considered perfect when a change of its temperature caused no motion of the revolving mirror.

In a test made at the Stevens Institute at a later date, it was found that changes of one-hundred-thousandth of an inch in the length of a bar of iron could be measured with the device.

Saxton's extraordinary fertility of invention is made manifest by the diversity of his mechanical productions during his residence in London, where his activities were not channelled in any specific direction by the nature of his employment. Among the other useful things he developed at this time were a fountain pen and a propelling pencil. He also designed and constructed the apparatus Wheatstone used to conduct his famous experiment of measuring the velocity of electricity in its passage over a long wire.

However, it is not to be supposed that his whole time was devoted to fidgetting with gadgets. There were times when he became absorbed in the solution of scientific problems. He performed a whole series of experiments on the adhesion of liquids. Another series was directed toward the location of the interior magnetic poles. He arrived at the conclusion that the magnetic polarity of the earth is deeply situated in the interior and that the magnetism of the globe may be represented by a comparatively short magnet, the axis of which passes through the center of the globe. It is interesting to recall that Poisson arrived at the same conclusion by mathematical means a short time afterwards.

Saxton had been admitted to a small group of scientists who met informally each week. The famous Saturday Club formed in Washington by Joseph Henry was conceived in imitation of the English club to which his friend had belonged. It is not at all improbable that the discussions at these meetings provided Saxton with suggestions for making his experiments. The friends and acquaintances he made at these meetings permitted him to enlarge his social circle and to spread his reputation. It is evident that he was appreciated by these and other men with whom he was brought into contact for, in 1837, without any solicitation upon his part, he was offered the post of director of the printing machinery in the Bank of England. This was a flattering offer, but Saxton did not hesitate to decline it.

He had two reasons for not accepting the offer. In the first place, he felt he had been long enough absent from his own country, to which he was deeply attached. The second reason was, that he had received an offer at the opportune moment which secured his return to America. Dr. R. M. Patterson, director of the United States Mint, who had known Saxton during his residence in Philadelphia and had learned of his many achievements in England, offered him the post of curator and constructor of weighing apparatus in the mint in Philadelphia. This was an offer in keeping with his inclination to return home.

His arrival in England had been attended by regrettable circumstances, but his departure was marked by a mild outburst of festivity that showed he had won a place for himself in the hearts of Londoners. The eighteen prominent scientists and engineers who were members of his club gave him a farewell banquet at the fashionable Piazzi Coffee House, with the customary parting gifts. One of these gifts was a book on mechanics and the inscription on the flyleaf said it was "a token of the high estimation in which they hold him as a mechanician of the first rank, and a man of science generally."

Returning to Philadelphia in 1837, Saxton renewed his association with The Franklin Institute and, in the same year, he was elected to the American Philosophical Society.

He remained for six years at the Mint, then situated at the corner of Juniper and Chestnut Streets in Philadelphia. During this time he introduced his ruling machine for copying medals, which Dr. Patterson was fond of demonstrating to distinguished visitors. He also constructed the large standard balances for use in the annual inspection of the assays and the verification of all the standard weights of the government assay and coining offices in the United States. The knife edges of the balance were made of the hardest steel available, and they turned upon plates of agate. This balance was so sensitive that, when loaded with fifty pounds they would turn with the addition of one-tenth of a grain, in other words with one-three-millionth of the load. This famous balance may still be seen in the Philadelphia Mint.

An invention that dates from about this time was Saxton's stove for burning anthracite. "Stone" or "Lehigh" coal, as it was called, was coming into more general use but many divergent ideas were advanced upon the best means of utilizing it. Saxton went beyond the mere design of the stove. There was a considerable difference of opinion upon the amount of draft required for the appropriate consumption, but he provided a means by which the draft was regulated by the different expansions of two metals, in the manner of a thermostat, by which a valve was controlled to maintain an equable predetermined temperature.

While employed at the Mint, Saxton displayed with what keen interest he followed developments in science. In the year 1839, Dr. John F. Frazer published his translation of Daguerre's description of his photographic process in the Institute's JOURNAL. This provoked a great amount of activity among the members, but Saxton was the first to produce practical results. He set about equipping himself with a complete photographic outfit. His camera was constructed from a cigar-box, with an ordinary burning glass for lens. Leaning out of a window on the north side of the Mint, he took a photograph of the Central High School with an exposure of about one hour. The exact date of the achievement is sometimes given as October 16, 1839, but

this cannot be verified. The silver photographic plate, $2\frac{1}{4} \times 2\frac{3}{4}$ inches was presented to the Historical Society of Pennsylvania. Thus it would appear that if Saxton was not the first to take a photograph in America, he was first to make an outdoor photograph.

During his times in Philadelphia Saxton was brought into intimate association with Alexander Dallas Bache who, in 1843, was appointed superintendent of the Coast Survey. This office included superintendence of all the weights and measures of the Government. One of Bache's first moves was to induce Saxton to accept charge of all construction of the standard balances, weights, and measures to be presented to the various States in order to insure uniformity in this field throughout the country. Saxton accepted this position and moved to Washington, where he was to spend the remainder of his life.

Soon his work expanded to encompass all the complicated instruments employed in the operations of the Coast Survey. Among the instruments he devised were some that exhibited a brilliant knowledge of mechanics. Only one or two of these can be mentioned. A necessary apparatus was one that would measure the rise and fall of the tides. The instrument devised by Saxton kept an hourly record of the changes. He also adapted his reflecting pyrometer for use in calibrating the measuring rods that would retain an unvarying length under variations of temperature. These rods were required for measuring the base lines and lines of verification in the survey. It is said that he was enabled to gain such a degree of accuracy with his improved methods, that measurements taken under different conditions of temperature over a distance of five miles would not show a variation in excess of $\frac{1}{4}$ inch.

A particularly ingenious contrivance was one for graduating the members of angular instruments. This was designed by him and executed from his drawings by another mechanic. It was a complex device comprising a small turbine wheel driven by water drawn from the house supply; a wheel and pulley that transmitted the power to a large wheel turning an axis tangent to a large circle by means of which a smaller circle could be divided. The automatic movements of a series of cams converted the rotary into vertical and horizontal motions that were recorded by a marking device.

At the meeting of the American Association for the Advancement of Science held in Baltimore in 1858, Saxton gave an account of the principal applications he had made with his reflecting pyrometer, which he had improved. One of the most notable of these applications was to indicate the magnetic dip and to magnify the motion of the axis of an aneroid barometer. This had been installed in the Girard College Magnetic Observatory, and is reported to have been so delicately sensitive that it responded to the very small changes in atmospheric pressure caused by the opening of a door, or the passage over the roof of a gust of wind.

A modification of the revolving mirror consisted of fastening a small mirror obliquely to a revolving axis. A ray of light when thrown upon the mirror, to which sufficient velocity was given, would be projected in the form of a large circle of light upon a prepared screen. When the ray of light came from the charcoal terminals attached to a galvanic battery, the resultant circle of light had a "dotted" appearance, indicating the alternating nature of the electric current.

It may not be generally known that the method of sealing by a flexible metal, instead of wax, was invented by Saxton. The need for a substitute for wax was drawn to his attention when the seals on some official documents melted in the mail as they passed through a country with a hot climate. He foresaw its value for attaching trade-marks to goods and other uses.

When the National Academy of Science was established by Act of Congress in 1863, Saxton was named as one of the original fifty members. This is proof, if further proof is needed, of the eminence he had gained as a designer and constructor of scientific instruments. It is not claimed for him that he was prominent in research. That was not his field. Nor was he merely a practical man who became expert through observing the operations devised by others. Rather, his was a mind that formed clear and definite conceptions of the physical principles involved and employed a controlled imagination in applying them to the problems awaiting solution.

While in Washington he gratified his fondness for an outdoor life by making long excursions on the water and in the nearby woods. He never lost his interest in fishing. During his walks through the woods and fields, he made an extensive collection of pre-historic implements and geological specimens.

In 1850, he married Mary H. Abercrombie, of Philadelphia, a lady some years younger than himself, but one who proved to be sympathetic to his nature and interests.

The later years of Saxton's life were clouded with suffering. In 1856, he suffered a cerebral hemorrhage which left him partially paralyzed. Ten years later another attack affected his speech. The difficulty in articulation gradually grew worse, and for the last two years of his life he was unable to communicate with his friends.

Those who knew him intimately spoke of him as a tall, robust man of modest character who, while he enjoyed the society of his fellow men, did not display his rich mental powers except within the circle of his most intimate friends. It might be interesting to attempt to determine what effect was produced upon his character by the direction of the energy and the prevailing thought during the times in which he lived, but that is beyond the range of this study. Let it suffice to say he appears to have adapted himself completely to the intellectual environment. He had the good fortune, denied to many men of inventive

minds, of being neither behind nor in advance of his age, but was in close harmony with it. He was conspicuously realistic and, in consequence, he did not plague his contemporaries with premature projects for which suitable conditions did not exist, nor did he retard progress by the advocacy of exploded theories under new forms. His inventions were based upon established principles, and were positive additions to human power and efficiency.

In 1935 the Institute Museum was able to display a rare assembly of objects made by or associated with Saxton, that were lent by his grandson, Joseph Saxton Pendleton, of Calcium, Pa. These served to revive an interest in the man whose reputation was almost unknown to the present generation, but who had been honored by his contemporaries for sterling work.

New TV Tube Developed for Use in Naval Aircraft.—A television tube consisting of transparent flat plates has been developed by Willys Motors for the navy in connection with a long-range program for simplifying aircraft instruments. These would both be television picture tubes. As now envisioned, one instrument would be a semi-circular plate mounted vertically and directly in front of the pilot. It would be transparent and thus would not interfere with the pilot's vision during flight. Altitude, speed and attitude of the aircraft would be shown on this plate, and physical features such as mountains, which the pilot sees during contact flight, would be depicted artificially. This instrument would tell the pilot all he needs to know to fly the aircraft about its three axes: pitch, roll and yaw. The second instrument would consist of a round plate mounted below the first, just inside the cockpit rim. Broad physical features of the earth below would be depicted by analogy, and the appearance would be somewhat similar to that of a radar map. Other information necessary for navigation or traffic control would also be shown on this instrument. By means of calibrations around the rim, the number of miles to the pilots base, fuel remaining, and similar information would be shown in a way that makes the information easy to read and assimilate. An aim of the Navy's long range program is to reduce the control system to two basic controls: a control stick and a throttle. Six switches would be used to select specific types of information needed for take-off, landing, for traffic control, and other specific situations. It is expected that the first experimental aircraft using this instrument and control system will be flown about 1958.

Loss of an Associate Editor.—ALBERT FRANCIS ZAHM, an associate editor of the JOURNAL OF THE FRANKLIN INSTITUTE and an American pioneer in aviation, died on July 23, 1954, at the age of 92.

A native of New Lexington, Ohio, Dr. Zahm attended Notre Dame University, receiving his Bachelor's degree in 1884 and his Master's degree in 1885. In 1892 he was awarded a Master of Engineering degree at Cornell University, and he obtained his Ph.D. at Johns Hopkins University in 1898. He received many academic honors, including Notre Dame University's Laetare Medal in 1925 and Villanova College's Mendel Medal in 1930.

Dr. Zahm taught mathematics and mechanics at Notre Dame University from 1885 to 1892; he served on the faculty of the Catholic University of America from 1895 to 1908; and he directed the Aerodynamical Laboratory of the United States Navy from 1917 until November of 1929. At that time he became the first Chief of the Congressional Library's newly organized Aeronautics Division and the first incumbent of its Chair of Aeronautics. He served in this position until his retirement in 1946.

Dr. Zahm was the author of several major works on various phases of aeronautical science and the inventor of a number of mechanical devices that forwarded the cause of aviation. During the years 1912 to 1948, Dr. Zahm published 22 of his papers in this JOURNAL.

The Institute mourns the passing of this pioneer in aviation, who contributed so materially to our JOURNAL.

A POSTULATIONAL APPROACH TO ELECTROMAGNETISM

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

"The first time that a French reader opens Maxwell's book, a feeling of *malaise* and even distrust at first mingles with his admiration. Only after long acquaintance and much work does this feeling vanish. For some eminent minds, the feeling never disappears.

"The English scientist does not try to construct a single, definite, well-ordered structure: he seems rather to raise a great number of provisional and independent houses, among which communication is difficult and sometimes impossible."

H. Poincaré[3]

"To a mathematician of the school of Laplace and Ampère, it would be absurd to give two distinct theoretical explanations of the same phenomenon, and to maintain that these two explanations are simultaneously valid. To a physicist of the school of Thomson or Maxwell, there is no contradiction in representing the same phenomenon by two different models. Moreover, the complication thus introduced into science never shocks the Englishman: for him it adds the charm of variety. . . . Thus in English theories we find those inconsistencies, those incoherencies, those contradictions which we are forced to judge severely because we seek a rational system, whereas the author has sought to present only a work of imagination."

P. Duhem[4]

"We have still in front of us the important task of deriving Maxwell's equations in such a way as will not contradict the atomic theory of electricity . . . when it [the task] is completed we shall find that, by one of those ironies of history, we shall have to a large extent returned to the views of Maxwell's predecessors and contemporaries, which he unwisely rejected."

A. O'Rahilly[5]

1. INTRODUCTION

A few definitions, chosen more or less at random from the literature of electromagnetism, are as follows:

(*a*) The *electric field strength* **E** at a given point P is equal to the force per unit charge which acts on a charged particle placed at P.

(*b*) The *voltage* V between two points a and b in an electric circuit is equal to the difference in potential between the two points:

$$V_{ab} = \varphi_a - \varphi_b.$$

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[3] H. POINCARÉ, "Electricité et optique," Paris, Carre et Naud, 1901, p. III.

[4] P. Duhem, "La théorie physique," Paris, Chevalier & Riviere, 1906, p. 129.

[5] A. O'Rahilly, "Electromagnetics," London, Longmans, Green & Co., 1938, p. 180.

(*c*) The *magnetomotive force* around a closed path is equal to the total conduction current linking that path:

$$MMF = \int \mathbf{J} \cdot d\mathbf{A}.$$

Are these statements true or false? Most students and some professors would undoubtedly say "true"; but actually, all three statements are false in general and apply only in certain special cases. Does this not indicate a weakness in the customary development of electromagnetic theory? Maxwell (1)[6] himself was aware that his treatise did not constitute a logical whole, yet he was never able to remedy the defect. Subsequent investigators have added further complications to Maxwell's theory but have not eliminated the looseness and ambiguity.

Electromagnetic theory is a peculiar subject. The peculiarity resides not so much in the stratification—superposed layers of electrostatics, magnetostatics, steady currents, and time-varying fields—as in the failure that has attended all attempts to weld these layers into a logical whole. The lowest layer, electrostatics, defines certain concepts, such as $\mathbf{E}$, $\mathbf{D}$, φ, in a way that is generally satisfactory only for the static case. Yet the attempt is made to force these specialized definitions into the higher strata, with *ad hoc* modifications when necessary. The student, in looking through his textbooks on electromagnetics, can find *general* definitions only with difficulty, if at all; and even the most advanced treatises fail to present a rigorously logical development of the subject.

In the following paragraphs, we examine the possibilities of formulating classical electromagnetism in a logical manner. This outline might be used, for instance, in writing an advanced textbook. Much of the material is obvious to anyone who has thought about the subject, but the inconsistencies and redundancies still existing in modern books support the belief that the present study is needed. In the final section of the paper, we compare the postulational approach for the classical theory with that for the new electrodynamics (2).

2. MAXWELL'S EQUATIONS

The essence of classical electromagnetism is contained in the four equations of Maxwell:

$$\left.\begin{aligned} \operatorname{curl} \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \\ \operatorname{curl} \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\ \operatorname{div} \mathbf{B} &= 0, \\ \operatorname{div} \mathbf{D} &= \rho. \end{aligned}\right\} \tag{1}$$

[6] The boldface numbers in parentheses refer to the references appended to this paper.

Do not the equations themselves constitute a sufficient basis for electromagnetism, without any further definitions or postulates? Lindsay and Margenau (3) say,

"In a very real sense, therefore, these equations may be said to constitute a *definition* of **E** and **H**."

But this cannot be true. Maxwell's equations are incomplete in at least two ways:

(i) They do not completely define the five field vectors **E**, **D**, **B**, **H**, **J**.
(ii) They do not specify the force on a charged particle.

It is well known that the specification of the curl *or* the divergence does not completely specify a vector. So even if one could assume that the quantities on the right side of Eq. 1 are known, he still would not have definitions of **H**, **E**, **B**, and **D**.

Additional equations may be introduced:

$$\mathbf{D} = \epsilon\mathbf{E}, \quad \mathbf{B} = \mu\mathbf{H}, \quad \mathbf{J} = \sigma\mathbf{E}, \tag{2}$$

where ϵ, μ, and σ are scalar constants. Substitution into Eq. 1 gives

$$\left.\begin{aligned} \operatorname{curl}\mathbf{H} &= \sigma\mathbf{E} + \epsilon\frac{\partial\mathbf{E}}{\partial t}, \\ \operatorname{curl}\mathbf{E} &= -\mu\frac{\partial\mathbf{H}}{\partial t}, \\ \operatorname{div}\mathbf{H} &= 0, \\ \operatorname{div}\mathbf{E} &= \rho/\epsilon. \end{aligned}\right\} \tag{1a}$$

Equation 1*a* applies only to a *linear*, *homogeneous*, *isotropic*, *time-invariant* medium. And even in this special case, Maxwell's equations do not constitute a complete basis for electromagnetism, since they do not specify a method for determining ϵ, μ, σ nor do they give any information about the force on charged particles. We conclude, therefore, that in a logical treatment of electromagnetism, Maxwell's equations must be preceded by definitions.

3. FUNDAMENTALS

The usual concepts of mechanics, including *length*, *mass*, *time*, and *force* are assumed to be known. As a basis for any formulation of electromagnetism, we must also have definitions of electric charge and current. These concepts will be defined *operationally* (4). That is, a definition will give either

(*a*) A description of how the quantity can be measured experimentally, or

(*b*) An equation that relates the new concept to concepts previously defined.

As the fundamental concept of electromagnetism, we take the *electric charge* Q. Frequent reference will be made to forces between charges, but such statements should not be construed as indicating a microscopic theory. We shall deal throughout with aggregates of charge, never with single electrons.

I. Definition of Charge

A particle may be acted on by forces of *mechanical origin*—by pushes and pulls communicated by rods or strings, by air currents, by inertial and frictional effects. It can also be acted on by *gravitational* forces. If all these forces on a dielectric particle are eliminated and still a force remains, we say that it is caused by *electric charges*.

By means of a torsion balance, Coulomb showed that the magnitude of the electrostatic force between two charges is

$$F = K\frac{Q_1Q_2}{r^2}, \tag{3}$$

where K is a constant that depends on the units and on the medium. Equation 3 may be used as an operational definition of the concept, *electric charge* Q.

Take three particles with charges Q_1, Q_2, Q_3. Using a torsion balance and keeping the distance r fixed, we find

$$F' = K\frac{Q_1Q_3}{r^2}, \qquad F'' = K\frac{Q_2Q_3}{r^2}.$$

Q_2 is adjusted so that $F' = F''$. Then $Q_1 = Q_2 = Q$. We now measure the force between Q_1 and Q_2:

$$F = K\frac{Q^2}{r^2}$$

or

$$Q = (F/K)^{\frac{1}{2}}r. \tag{3a}$$

Equation 3*a* defines *charge* in terms of the known concepts of force and distance.

II. Definition of Charge Density

Charge density ρ is defined by the equation

$$\rho = \lim_{v\to 0}\left(\frac{Q}{v}\right). \tag{4}$$

The limit is, of course, a mathematical fiction. Physically, the volume is imagined to be reduced to a small but macroscopic size which still contains a large number of electrons (say 10^{10}). The function $\rho(u^1, u^2, u^3, t)$ is considered to be continuous and with continuous first derivatives at ordinary points.

III. Definition of Current Density

Current density $\mathbf{J}$ is defined as

$$\mathbf{J} = \rho_+\mathbf{v}_+ + \rho_-\mathbf{v}_-, \tag{5}$$

where the subscripts refer to the sign of the charge. If only the electrons move, as in a metallic conductor,

$$\mathbf{J} = \rho_-\mathbf{v}_- = -N|Q_e|\mathbf{v}_-, \tag{5a}$$

where

N = number of electrons per unit volume,
$|Q_e|$ = magnitude of electronic charge.

The velocities in Eqs. 5 and 5*a* are ordinarily measured with respect to the conductor. But here, as throughout classical electromagnetism, velocity is ambiguous.

IV. Definition of Current

The current through a surface S is defined as

$$I = \int_S \mathbf{J}\cdot d\mathbf{A}. \tag{6}$$

V. Definition of Permittivity

Consider a parallel-plate capacitor connected to a battery. Measure the charge on one of the plates, first with vacuum dielectric, secondly with a given dielectric. The ratio of the two values of Q is called the relative permittivity (ϵ/ϵ_0) of the given dielectric, or

$$\epsilon = \epsilon_0(Q/Q_0), \tag{7}$$

where ϵ_0 is an arbitrary constant which depends only on the system of units. Equation 7 defines a property of a material, and this property is called *permittivity.*

VI. Definition of Permeability

Consider a toroidal core made of the material to be tested. A uniform winding on the core is connected to a sinusoidal electric source of

constant frequency. The relative permeability (μ/μ_0) of the core may be defined as the ratio of the magnetizing currents obtained with the material core and with a vacuum core, or

$$\mu = \mu_0(I/I_0). \tag{8}$$

Equation 8 defines a property of a material, this property being called the *permeability*.

I. Postulate of Retardation

It is a well-known experimental fact that electromagnetic disturbances do not arrive instantaneously at a distant point but are retarded by a time interval equal to l/c. Here c is a constant that depends on the medium. In a vacuum, $c = 2.99792 \times 10^8$ m sec^{-1}. In dissipative media, the form of a transient may be so completely distorted that the concepts of phase velocity and group velocity are practically meaningless. Nevertheless, we shall postulate that all time relations obtained experimentally at very small distances can be employed at large distances merely by replacing t by $(t - l/c)$.

TABLE I.—*Fundamentals.*

I. Definition of *charge:*

$$Q = (F/K)^{\frac{1}{2}}r.$$

II. Definition of *charge density:*

$$\rho = \lim_{v \to 0} \left(\frac{Q}{v}\right).$$

III. Definition of *current density:*

$$\mathbf{J} = \rho_+\mathbf{v}_+ + \rho_-\mathbf{v}_-.$$

IV. Definition of *current:*

$$I = \int_S \mathbf{J} \cdot d\mathfrak{A}.$$

V. Definition of *permittivity:*

$$\epsilon = \epsilon_0(Q/Q_0) \text{ for parallel-plate capacitor.}$$

VI. Definition of *permeability:*

$$\mu = \mu_0(I/I_0) \text{ for toroidal coil on a.c.}$$

I. Postulate of retardation.

A summary of this section is given in Table I. The definitions of this section are a necessary preliminary to any formulation of classical electromagnetism, either of the type of Section 4 or that of Section 5.

4. DEFINITIONS OF E AND B

Consider first the problem of finding a definition of **E** that will apply in both electrostatics and electrodynamics. The usual electrostatic definition is

$$\mathbf{E} = \mathbf{F}/Q. \tag{9}$$

But this equation is inadequate for general use:

(*a*) In free space, the operational experiment (**4**) associated with Eq. 9 is easily imagined and can even be performed. But in a solid dielectric, the operational significance of the definition becomes questionable.

(*b*) Equation 9 does not limit the magnitude Q of the test charge, though a large test charge will distort the original field that is to be measured. This difficulty is eliminated by the modification,

$$\mathbf{E} = \lim_{Q \to 0} \left(\frac{\mathbf{F}}{Q} \right). \tag{9a}$$

But obviously the limiting process cannot be taken literally, since the minimum possible charge is that of an electron. With either the microscopic or the macroscopic interpretation, Eq. 9*a* involves troublesome explanations.

(*c*) The usual equation for the force on a moving charge is

$$\mathbf{F} = Q[\mathbf{E} + \mathbf{v} \times \mathbf{B}], \tag{10}$$

and this equation may be employed as a general definition of **E**. Lorentz considered **v** as the velocity of the test charge with respect to the aether. But since the aether has been abandoned, **v** seems to be the velocity of Q with respect to the magnetic field, If **B** is produced by several current-carrying conductors moving with respect to each other, however, it is not clear (at least, without further explanation) what **v** is.

Similar difficulties are encountered in attempting to define (**5**) the magnetic flux density **B**. We may imagine a current element $I\,d\mathbf{s}$, placed at point P where **B** is to be measured. The force on the element is

$$d\mathbf{F} = I(d\mathbf{s} \times \mathbf{B}), \tag{11}$$

and the equation defines **B**. All the weaknesses of the previous definitions appear, plus the added troubles associated with a fictitious, isolated current element.

In particular, Eq. 11 raises the question of the absolute velocity of the charges which constitute the current. Evidently, an explanation is needed about this velocity—is it with respect to the aether, with respect

TABLE II.—*Outline of Electromagnetic Theory.*

DEFINITIONS

(1) Definition of φ:

$$\varphi = k \int \frac{[\rho]}{r} d\mathcal{V}.$$

(2) Definition of $\mathbf{A}$:

$$\mathbf{A} = k' \int \frac{[\mathbf{J}]}{r} d\mathcal{V}.$$

(3) Definition of $\mathbf{E}$:

$$\mathbf{E} = -\operatorname{grad} \varphi - \frac{\partial \mathbf{A}}{\partial t}.$$

(4) Definition of $\mathbf{B}$:

$$\mathbf{B} = \operatorname{curl} \mathbf{A}.$$

(5) Definition of $\mathbf{D}$:

$$\mathbf{D} = \epsilon \mathbf{E},$$

where $\epsilon = \epsilon_0(Q/Q_0)$ for a parallel-plate capacitor.

(6) Definition of $\mathbf{H}$:

$$\mathbf{H} = \mathbf{B}/\mu,$$

where $\mu = \mu_0(I/I_0)$ for a toroid excited with sinusoidal a.c.

POSTULATES

(1) Conservation of charge:

$$\operatorname{div} \mathbf{J} = -\frac{\partial \rho}{\partial t}.$$

(2) Force on charge:

$$\mathbf{F} = Q[\mathbf{E} + \mathbf{v} \times \mathbf{B}],$$

where $\mathbf{v}$ is the velocity of Q with respect to the magnetic field.

DEDUCTIONS

(1) From Definition 4,

$$\operatorname{div} \mathbf{B} = 0.$$

(2) From Definitions 3 and 4,

$$\operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

(3) From Definitions 1, 2, 3, and Postulate 1,

$$\operatorname{div} \mathbf{D} = \rho,$$

(4) And

$$\nabla^2 \varphi + \rho/\epsilon = \epsilon\mu \frac{\partial^2 \varphi}{\partial t^2}.$$

(5) From Definitions 2, 3, 4,

$$\operatorname{curl} \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t},$$

(6) And

$$\bigstar \mathbf{A} + \mu \mathbf{J} = \epsilon\mu \frac{\partial^2 \mathbf{A}}{\partial t^2},$$

to the wire, or with respect to the magnetic field? One may introduce the Einstein special theory of relativity, which is based on a naive faith in the infallibility of Maxwell's equations. But if an electrodynamic is itself based on the fundamental principle that absolute velocity is meaningless (as was done by Ampère, Gauss, Ritz, and others), then no special relativity crutch is needed (2).

In this section, we have considered the possibility of a logical development of electromagnetism beginning with a minimum number of definitions, to be followed by Maxwell's equations. The preliminary definitions seem to offer almost insuperable difficulties. In view of this result, it is perhaps better to abandon the attempts made in this section and to try an entirely different approach. A postulational development based on scalar and vector potentials is outlined in the following sections.

5. FURTHER DEFINITIONS AND POSTULATES

Starting with the preliminary definitions of Table I, we proceed to the logical development of electromagnetism summarized in Table II. There are six definitions, two postulates, and a set of deductions. The first definition specifies the scalar potential in terms of a distribution of charge ρ. The integral is taken over all space, and the charge density

$$[\rho] = \rho(u^1, u^2, u^3, t - r/c)$$

is retarded in accordance with Postulate I. A similar definition gives the vector potential **A**. The square brackets indicate retardation, according to the usual Lorenz notation. The scalar constants k and k' depend on the medium and on the system of units.

Note that the foregoing definitions are general, in as much as they are not limited to electrostatics or magnetostatics. Unfortunately, however, they are limited to *linear* media, since only with a linear system can we use superposition or integration. Also, c is questionable unless the medium is linear and lossless. There seems to be a widespread misconception as to the universality of electromagnetic equations. Even if one assumes that the Maxwell equations in differential form are universally true, he can do practically nothing with them until he introduces further definitions or manipulations that limit generality. Thus there seems to be little disadvantage in restricting ourselves from the start to media that are linear, homogeneous, isotropic, time-invariant, and lossless.

We next define the vector **E** in terms of the potentials (Definition 3, Table II). Similarly, the vector **B** is defined in terms of the vector potential (Definition 4). Definitions 5 and 6 allow the evaluation of k and k' in Definitions 1 and 2. In the rationalized mks system,

$$k = 1/4\pi\epsilon, \qquad k' = \mu/4\pi.$$

Only two postulates are needed: conservation of charge, and the equation for force on a moving charge (Table II).

6. DEDUCTIONS

From the six definitions and two postulates (plus the material in Table I), one should be able to deduce all of electromagnetic theory. Using Definition 4 and a familiar vector identity, we obtain

$$\text{div } \mathbf{B} \equiv \text{div curl } \mathbf{A} \equiv 0$$

or

$$\text{div } \mathbf{B} = 0, \tag{12}$$

which is one of Maxwell's equations.

From Definition 3 and another vector identity,

$$\text{curl } \mathbf{E} = -\text{ curl grad } \varphi - \frac{\partial}{\partial t}(\text{curl } \mathbf{A}).$$

But curl $\mathbf{A} = \mathbf{B}$ and curl grad $\varphi \equiv 0$, so

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \tag{13}$$

From Green's theorem (6) and Definition 1, we obtain the important differential equation,

$$\nabla^2 \varphi + \rho/\epsilon = \epsilon\mu \frac{\partial^2 \varphi}{\partial t^2}. \tag{14}$$

Thus,

$$\text{div grad } \varphi + \rho/\epsilon = \epsilon\mu \frac{\partial^2 \varphi}{\partial t^2},$$

and by Definition 3,

$$-\text{ div } \mathbf{E} + \rho/\epsilon = \frac{\partial}{\partial t}\left[\text{div } \mathbf{A} + \epsilon\mu \frac{\partial \varphi}{\partial t}\right]. \tag{15}$$

Using Postulate 1 and Definitions 1 and 2,

$$\text{div } \mathbf{A} = \frac{\mu}{4\pi} \int \frac{\text{div } [\mathbf{J}]}{r} d\mathcal{V} = -\frac{\mu}{4\pi} \int \frac{1}{r} \frac{\partial [\rho]}{\partial t} d\mathcal{V}$$
$$= -\epsilon\mu \frac{\partial \varphi}{\partial t}.$$

Thus the bracket in Eq. 15 is zero, and

$$\text{div } \mathbf{D} = \rho. \tag{17}$$

From Green's theorem and Definition 2, we obtain the partial differential equation (7),

$$\bigstar \mathbf{A} + \mu \mathbf{J} = \epsilon\mu \frac{\partial^2 \mathbf{A}}{\partial t^2}. \tag{18}$$

Thus,

$$\text{grad div } \mathbf{A} - \text{curl curl } \mathbf{A} + \mu \mathbf{J} = \epsilon\mu \frac{\partial^2 \mathbf{A}}{\partial t^2},$$

and from Definitions 3 and 4,

$$\text{grad div } \mathbf{A} - \text{curl } \mathbf{B} + \mu \mathbf{J} = -\epsilon\mu \frac{\partial \mathbf{E}}{\partial t} - \epsilon\mu \text{ grad } (\partial\varphi/\partial t).$$

But $\text{div } \mathbf{A} = -\epsilon\mu \dfrac{\partial \varphi}{\partial t}$, so

$$\text{curl } \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}. \tag{19}$$

Equations 12, 13, 17, 19 are Maxwell's equations, which are here derived from the definitions and postulates of Tables I and II, without use of Section 4. All of classical electromagnetic theory is obtainable directly from Maxwell's equations, so the whole subject rests on the postulation system of Tables I and II. In specific applications, the vectors $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}$ are usually obtained from the potentials φ and $\mathbf{A}$. These potentials may be evaluated from Definitions 1 and 2 or from the partial differential expressions, Eqs. 14 and 18. The vector quantities are then found by applying Definitions 3, 4, 5, and 6.

7. THE NEW ELECTRODYNAMICS

An alternative to Maxwell's theory is based on the work of Ampère, Gauss, and Ritz. In this form of electrodynamics, the physically measurable quantities *force* and *charge* are emphasized, and the fictitious magnetic field is entirely abandoned. The result is a logical system with only a few definitions and postulates, free from the ambiguities of the Maxwell theory.

TABLE III.—*The New Electrodynamics.*

I. Definition of *charge:*

$$Q = (F/K)^{\frac{1}{2}} r.$$

II. Definition of *charge density:*

$$\rho = \lim_{v \to 0} \left(\frac{Q}{v} \right).$$

III. Definition of *permittivity:*

$$\epsilon = \epsilon_0 (Q/Q_0).$$

I. Postulate of retardation.

II. Postulate of force (2)

$$\frac{\mathbf{F}}{Q_2} = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2 [1 - \tfrac{3}{2}\cos^2\theta] - \mathbf{a}_a \frac{Q_1}{4\pi\epsilon c^2 r}\frac{d}{dt}v(t - r/c) - \mathbf{a}_r \frac{1}{4\pi\epsilon}\frac{\partial}{\partial r}\left[\frac{1}{r}Q_1(t - r/c)\right].$$

Table III lists the entire set of definitions and postulates needed for the new electrodynamics (2). Postulate II expresses the force per unit charge, on a macroscopic charge Q_2, caused by a charge Q_1 which may have any velocity with respect to Q_2 and any acceleration. For a continuous distribution of charge,

$$Q_1 = \rho d\mathcal{V}$$

and the total force is obtained by integration.

The new electrodynamics may be expressed as a field theory if desired. The only field vector is $\mathfrak{F}$, where

$$\mathfrak{F} = \lim_{Q\to 0} (\mathbf{F}/Q_2). \tag{20}$$

The usual quantities, **E**, **D**, **B**, **H**, **J** and μ are eliminated. With stationary charges (electrostatics), $\mathfrak{F}$ become identical with the familiar **E** and a scalar potential may be introduced:

$$\mathfrak{F} = -\operatorname{grad}\varphi. \tag{21}$$

With moving charges, a vector potential is also needed. Evidently this form of electrodynamics is simpler and more closely related to the measurable quantities than is the Maxwell theory.

The new formulation is also free from the outstanding defects of the Maxwell theory. As mentioned previously, one of the troubles with classical electromagnetism is the ambiguity with respect to velocity. In the pre-Maxwellian work of Galileo, Newton, Ampère, and Gauss, *relative* velocities were used as a matter of course. But Faraday and Maxwell introduced the questionable idea of fields in a stationary aether, thus apparently employing the idea of *absolute* velocity. The concepts of current and current density are tainted by this vagueness. Even Definitions 1 and 2 of Table II are under suspicion, for they assume that we can always tell whether charges are classified as ρ or **J**. Physically, there are not two distinct quantities here but only one; and whether a given charge distribution is specified as ρ or **J** depends on what physical velocity is arbitrarily considered to be zero. The force between two charges depends on the velocity of one with respect to the other, not primarily on their velocities with respect to the aether, the laboratory,

or the observers. Two parallel streams of electrons moving at the same velocity exert no more force on each other than they would if they were stationary in the laboratory. Since the new electrodynamics employs only relative velocities, it does not encounter these difficulties nor does it need to be patched up by the introduction of a special theory of relativity.

To summarize, we feel that all special cases of electromagnetism, such as the stationary and quasi-stationary states, may well be derived from Maxwell's equations. But before these equations are stated, logical development requires that the vectors be defined. Even when the investigator carefully avoids the familiar fairy tales dealing with fictitious cavities and "wormholes" and the pushing of charged particles through solid but frictionless media, he still encounters trouble in defining the concepts (Section 4). The least objectionable approach that we could find to classical electromagnetics is summarized in Table II. And even that approach has trouble with the concept of velocity and with the generality of the results. Another possibility is the new electrodynamics, which offers interesting possibilities that merit further study.

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New Theory on Lightning.—Lightning, which kills 400 people each year in the U. S. and sparks ten per cent of our forest fires, is still feared by humans and not fully understood by scientists. The actual mechanics of a flash of lightning are fairly well understood but the mystery still remains, "What causes the electricity in the first place?". At the present time it is generally believed that the electrification is caused by the falling of charged precipitation particles. But, how these precipitation particles become electrically charged is still very much in doubt. There are at present seven different mechanisms that have been advanced to explain the electrification of precipitation. Dr. Bernard Vonnegut, who pioneered in rain-making and is a physicist at Arthur D. Little, Inc., is proposing a new theory of electrification that is not dependent upon precipitation and states that electrification occurs long before precipitation has occurred. He presented his theory at the American Meteorological Society, on January 26, at New York University. He explained how his new theory of electrification, if correct, may play a prominent part in the formation of rain.

It is a well-known fact that the charge in the lower atmosphere near the surface of the earth carries a slight positive charge. This positively charged air rises with strong updrafts to upper levels to form positively charged clouds. Fast negative ions from the upper atmosphere are attracted to the surface of a positively charged cloud and attach themselves to the cloud particles and become changed into ions of low mobility. These negatively charged particles are swept by downdrafts to the lower part of the cloud. This is a continuous process and after a length of time a strong negative charge is accumulated in the lower part of the cloud.

When this negative charge becomes sufficiently large it causes a discharge of fast positive ions from points on the earth's surface. These positive ions become attached to fine particles, always present in the air at low levels, and are thereby transformed into ions of low mobility. These positive ions are then carried up into the upper part of the cloud by updrafts, thus increasing the positive charge in the upper part of the cloud. This increase in the positive charge increases the negative charging rate, intensifying the negative charge on the lower part of the cloud. This results in a "bootstrap" electrification process and the charges become large enough to produce lightning.

Dr. Vonnegut is well known for his early research work concerning cloud-seeding, which he conducted at the General Electric Company, and for more recent work on cloud physics, which is being carried out at Arthur D. Little, Inc. In addition to his work in meteorology, Dr. Vonnegut has conducted research on the strength of glass, air pollution, and aeroplane instrumentation.

Petroleum Lifeline.—Medical researchers, with oil's help, have discovered a way to deliver a bigger radioactive blast at cancer of the pancreas. The method is to thread a fine polyethylene tubing around and through the cancer. The ends of the tubing are allowed to project outside the body after the surgical wound has been closed. Radioactive iodine in liquid form is then inserted in the tubing and the two ends filled with air or mercury and sealed. The radioactive iodine can be left in the patient until it loses its radioactivity, or it can be withdrawn if further surgery is needed. After treatment, the plastic tubing can be left in the body, since it causes no difficulty.

DETAILED EXAMPLE OF INTERCONNECTING PIECE-WISE SOLUTIONS

BY

GABRIEL KRON [1]

SYNOPSIS

Detailed description is given of all the steps involved in inverting on a small digital computer (such as the CPC) a special type of 63 × 63 matrix. This matrix may represent the linear equivalent circuit of the field equations of Maxwell, or the diffusion equation of a nuclear reactor, etc. Or it may represent the equivalent circuit of a non-linear partial differential equation for one particular situation. These networks have, however, the special property that each subdivision is separately grounded. In the example given, the physical system is torn apart into four parts, each part is solved independently of the others, then the solutions are interconnected. The inverse matrix is given also in a factorized form, more suitable than an actual inverse for many purposes. A simple numerical example is also worked out step-by-step.

INTRODUCTION

Solutions with Permanent and Growing Utility

Given a physical system shown in Fig. 1, described by the linear equations $\mathbf{I} = \mathbf{YE}$, where $\mathbf{Y}$ is a matrix with 63 rows and columns. All

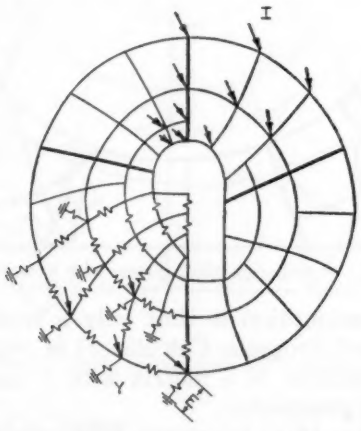

FIG. 1. Physical system with 63 variables.

the $\mathbf{I}$ are known and all $\mathbf{E}$ are unknown. (The physical system may be *non-linear*, but for one particular situation all the elements of $\mathbf{Y}$ may be known.) *The problem at hand is to invert the* $\mathbf{Y}$ *matrix;* or

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rather to find the inverse equations $\mathbf{E} = \mathbf{ZI}$, so that for *any* particular set of given $\mathbf{I}$ the unknown $\mathbf{E}$ can immediately be determined by simply multiplying the matrix $\mathbf{Z}$ by the vector $\mathbf{I}$.

Three assumptions are to be made concerning the nature of the solutions wanted:

1. It is assumed first of all that a large number of $\mathbf{E}$ are to be determined and not merely one or two, so that the availability of a permanent inverse matrix $\mathbf{Z}$ is preferred.

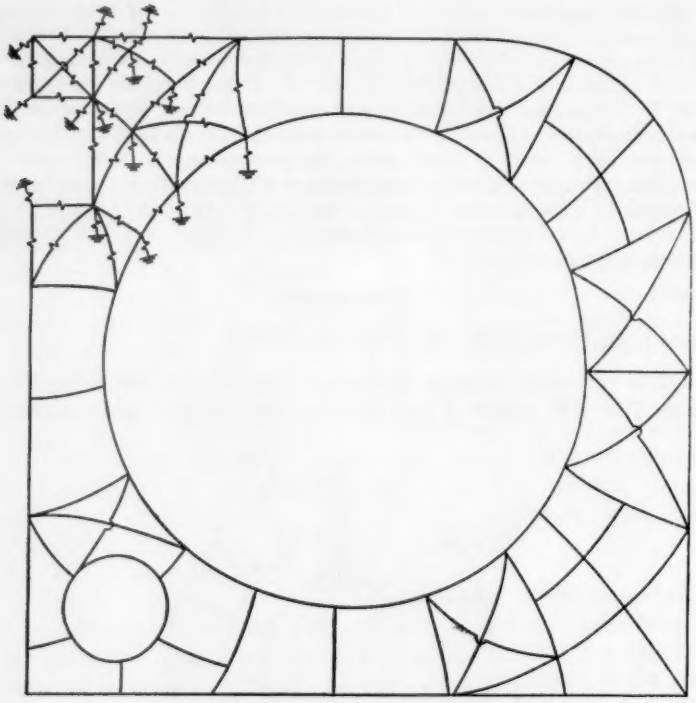


FIG. 2. Another physical system with 62 variables.

2. The second assumption is that only a small digital computer, such as a CPC (Card Program Calculator) is available on which the inversion or multiplication of a matrix with a maximum of about 16 rows and columns is practicable.

The inversion of a 16×16 matrix takes about one hour and ten minutes and to multiply together two such matrices takes about forty-five minutes. The ceiling number 16 is determined by the nature of the punch-cards, each of which can carry only eight numbers. Theoretically a CPC can invert a larger matrix than a 16×16, but the amount of work and complications increase as the cube of the number of rows and columns.

3. A third assumption will also be made in the present example, that it is intended to re-utilize the resultant inverse matrix **Z** of Fig. 1 with the **Z** of the structure of Fig. 2 for the construction of **Z** of a still larger physical system such as shown in Fig. 3.

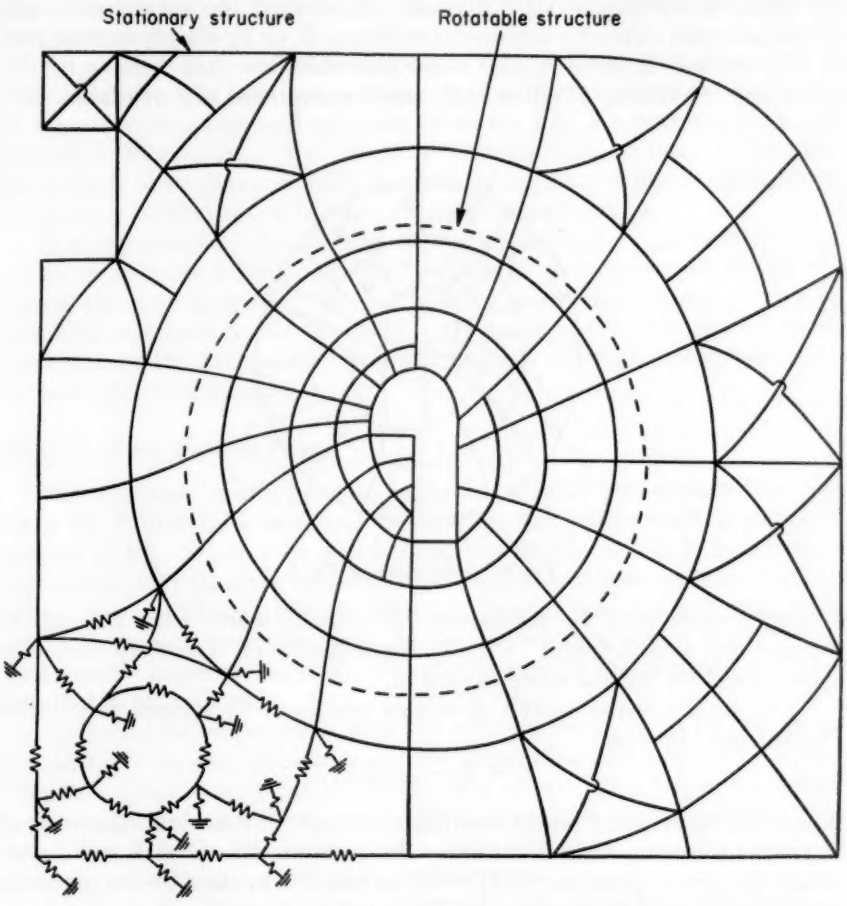


FIG. 3. Super-system with 125 variables.

Piece-Wise Solution of a System

The steps of solving a *complex* network by tearing it apart into a set of smaller networks are quite analogous to tearing a given *simple* network into its isolated coils (the "primitive" system). Namely, the equations of the latter are written by mere inspection, then the isolated coils are interconnected into the given network by a connection tensor

C. Anyone who understands these last steps knows already how to follow their large-scale analogue also.

The inversion of a large matrix on a small machine consists thus of tearing the given physical system apart into several smaller subdivisions (Fig. 4) and inverting the **Y** matrix of each subdivision separately as if the other subdivisions did not exist. Afterward the subdivisions are interconnected either by a connection tensor **C**, or by simply *rearranging* in the inverse **Z** some of the rows and columns that belong to the boundary variables. (When only small computers are available, this

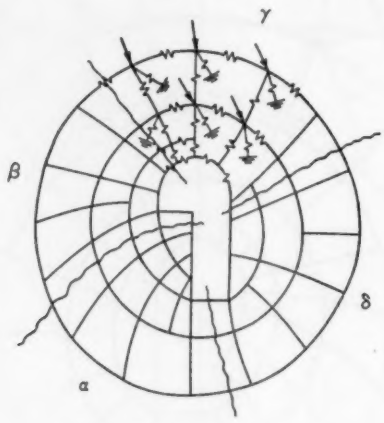


FIG. 4. Subdivision of Fig. 1.

rearrangement may be performed by clerical work, with the aid of scissors and scotch-tape.) Finally the superfluous boundary variables **i** are solved for by one more inversion.

The final inverse matrix **Z** comes out in a factorized form (the "factorized" inverse)

$$\mathbf{Z} = \mathbf{Z}_1 - \mathbf{Z}_2\mathbf{Y}_4\mathbf{Z}_3$$

where the component matrices altogether contain a far fewer number of non-zero elements than **Z** would. As a result, the final **E** are calculated (for every given set of **I**) with far less work, than in the presence of a conventional inverse matrix **Z**.

Re-Utilization of Calculated Inverse

One type of re-utilization of the calculated factorized inverse **Z** will be shown in a subsequent paper in which the present physical system will correspond to a rotatable structure that assumes 16 different angular positions within the stationary structure of Fig. 2. (The latter itself is represented by 62 equations $\mathbf{I} = \mathbf{YE}$.) One particular position is shown in Fig. 3.

In other words the actual problem undertaken in the present and in a subsequent paper is *the inversion of sixteen different matrices*, each with 125 rows and columns. The method of attack consists of inverting separately the rotating and the stationary parts. Afterward they are interconnected in 16 different positions, each interconnection, however, requiring the inversion of only a 16 × 16 matrix (as there are only 16 points of contact).

All sixteen **Z** matrices come out in such a form that they can be re-utilized outright for the construction of still larger factorized inverses of more complex physical systems, of which Fig. 3 represents only one portion. Moreover, if the shape or construction of one of the subdivisions is to be changed, only that particular inverse need be subjected to change, leaving much of the calculations unchanged.

A comparison of the method of tearing with the "partitioning" of matrices appears in reference 3.[2] There is no relation whatever between the two methods. In partitioning a *physical system* a host of new and unknown forces (variables) are set free, that are put to useful work during the inversion. In partitioning a *set of equations*, however, no new variables appear.

Purpose of the Present Paper

The purpose of the present paper is to show *step-by-step* how the 63 × 63 **Y** matrix is inverted by tearing the corresponding physical system of Fig. 1 into four parts, as shown in Fig. 4. It is hoped that the reader will be able to perform similar inversions without further help. For a philosophy of the steps the reader may consult references 1–4. Numerical examples and checks are worked out in reference 4. The present paper restricts itself to giving specific instructions, without detailed explanations.

It should be emphasized that the selection of a 16 × 16 matrix as a maximum is due to assuming the capabilities of the CPC (Card Program Calculator, built by IBM) as a norm. If a larger computer is available that can invert, say, a 60 × 60 matrix within a reasonable time, then exactly the same steps should be applied for the inversion of a matrix with about 240 rows and columns.

Similarly the basic steps are identical if the number of subdivisions is greater than four. Any difference that may arise is due to the manner of elimination of the more than 16 variables **i** occurring at the cuts. A subsequent paper will show one way how to handle eight subdivisions, in which there are 40 boundary variables **i** to eliminate.

On the other hand if no digital computer is available, only a desk calculator, or a slide-rule, then each subdivision should be small enough to require the inversion of only a 3- or 4-rowed matrix. All steps to

[2] A bibliography of numbered references is appended to this paper.

be given are equally valid, without any change, for the interconnection of such small-scale solutions also.

Limitations of the Example

To avoid any misunderstanding, it should be emphasized that the method to be shown is not valid for the inversion of an *arbitrary* matrix with 63 rows and columns. The method assumes the following limitations:

1. A physical system, or a model of it is given, not a set of equations (or difference equations).
2. It is the physical system that is torn apart, not the set of equations.
3. No mutual relations exist among the isolated, torn apart subdivisions. (In the general case this assumption is not a necessity. The absence of mutual coupling is assumed here only to simplify the presentation.)
4. The particular example worked out here assumes that each subdivision is separately grounded. If that is not true (for instance in the network representing Laplace's or Poisson's equation), *additional* steps are necessary that supplement the steps shown here (reference 1).
5. It is also assumed that the inverse of each subdivision does exist. In many mechanical problems that is not the case and further steps are needed.

Within each subdivision there may exist *asymmetrical* mutual impedances between each coil. Also each element of a matrix may contain complex numbers.

Types of Problems to be Worked Out

In particular, the present example may represent the following types of problems:

1. An electromagnetic field problem with losses, such as a cavity resonator or an induction heater, described by the two-dimensional field equations of Maxwell.
2. Neutron diffusion in a nuclear reactor.
3. Vacuum-tube circuit or other general circuit with currents impressed at junction points and in which each subdivision has at least one grounded point.
4. Radiation exchange in an enclosure.
5. Electrical power-transmission network with capacitors or leakages to ground.
6. Mechanical systems (or their electrical models) containing elements with *one* degree of freedom only (springs, masses, etc.).

It is a well-known fact that in the solution of *non-linear* systems by successive approximation, *the availability of even an approximate inverse matrix greatly speeds up the convergence of the solutions.* The circuit of the present paper thus may be assumed to represent also some non-linear partial differential equation or, say the non-linear radiations in a combustion chamber, at some particular guessed-at value of the dependent variables. A more detailed study of the solution of certain types of non-linear systems by diakoptic (tearing) methods will be given in another publication.

When the various subdivisions contain different parameters, it is possible to solve (invert) the system equations with all the parameters factored out. Such an *analytical solution* of complex physical systems with odd shapes will be shown in another publication.

PROCEDURE

Establishment of a Model

The electrical model of the physical system under consideration is shown in Fig. 1. The physical system has an elliptical outer periphery and an odd-shaped internal boundary and it is constructed out of four different materials, as shown by the four straight lines in Fig. 1. In order to make the present paper stand on its own feet, it will be assumed that there is *as yet* no intention to interconnect the elliptical system of Fig. 1 with that of Fig. 2. Hence the periphery of Fig. 1 contains no extra leads to interconnect it with another system. The extra leads can and will be added *after* the complete inversion.

In order to have more points near the discontinuities along the internal boundaries, a highly irregular net with 63 junction points is constructed. It is also assumed that the admittances $Y(= I/E)$ of each coil have already been calculated by known methods. It has to be remembered that:

1. A coil exists between several of the junctions and the ground, as shown in Fig. 1. These coils will usually be omitted on the drawings (so will the resistance representation of the other coils).
2. A known current I is impressed at every junction. (Some of the I may have zero value.) They also will not be shown most of the time.

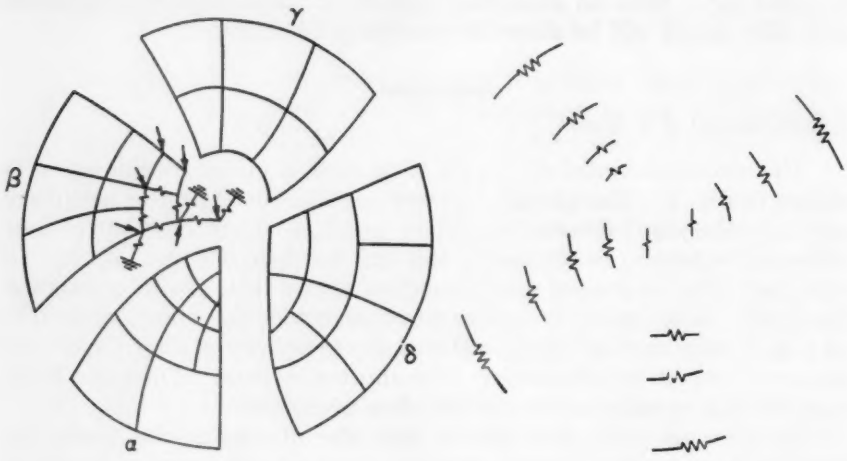
Tearing Apart into Four Subdivisions

Since the digital computer available is assumed to invert a maximum of 16×16 matrix, the network of Fig. 1 is subdivided into four parts, as shown in Fig. 4. One part contains 15 junctions, each of the other three parts contains 16 junctions. The four cuts separate each of the sixteen boundary cut coils into two arbitrary parts. (The currents in these sixteen cut coils are represented by **i** whose value is unknown.)

Removal of Cut Boundary Coils

The next step is to *remove all the sixteen coils that have been cut*, so that only the diminished network of Fig. 5 has to be inverted. (For a detailed explanation see reference 4.) Without this step each subdivision would require the inversion of a much larger matrix than a 16×16.

It should be noted from Fig. 5 that while the four subdivisions are left in their original location, the sixteen boundary coils are removed and laid temporarily aside. (The latter will be interconnected only *after* all the left-hand subsidivions have been.) *These removed coils are*



a) Primitive system of four subdivisions

b) Primitive system of cut boundary coils

FIG. 5. Removal of cut boundary coils.

still part of the "primitive" system. They may be looked upon to form a fifth subdivision, subject, however, to special handling.

Numbering of the Rows of **Y**

Considering subdivision α, shown separately in Fig. 6, it has 16 junction points, 16 vertical coils and 23 horizontal coils. The coils may be numbered in an arbitrary manner, but the order of junctions must follow a definite pattern, in order to facilitate the interconnection of subdivisions. In particular:

1. Those junctions that have no cut boundary coils for neighbors, are numbered first (in an arbitrary order) from 1 to 8.

2. Arrows are drawn along each cut. Here two cuts exist, hence two arrows.

3. The junctions along the cuts are numbered *in the direction of the arrows*. From 9 to 11 along one cut and from 12 to 16 along the other.

If it is definitely known that eventually it is intended to interconnect the present system to some other system along the outer periphery, then a slightly different numbering is followed. However, any type of

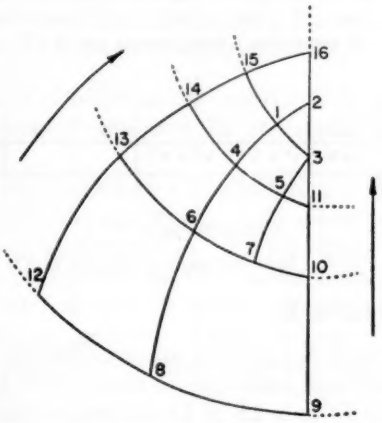


FIG. 6. Numbering of junctions in subdivision α.

future interconnection may be performed even with the present numbering. The only extra change will be a rearrangement in the order of the final rows and columns.

Admittance Matrix **Y**

The admittance matrix of subdivision α may be constructed as follows:

1. A square table with 16 rows and columns is constructed and numbered from 1 to 16, corresponding to the 16 junction points.

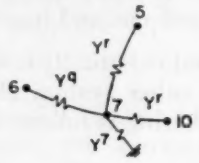
2. The diagonal elements contain the sum of all the horizontal and vertical admittances that adjoin the particular junction point.

3. In each row all *neighboring* junction points contain the corresponding negative **Y**'s that occur in the circuit.

For junction No. 7 the respective seventh row of **Y** is shown in Fig. 7. Each element may contain complex numbers. In general some or all of the coils of each subdivision may be mutually coupled asymmetrically. Also the vertical coil may be absent.

The **Y** matrix could have been established also by removing all interconnections, establishing a connection matrix **A** and finding **Y**′

from that of the primitive system by $\mathbf{A}_t\mathbf{Y}\mathbf{A}$. The result would be identical with that found by the scheme of Fig. 7.



a) Admittances entering seventh row of Y^α

4	5	6	7	8	9	10	11
	$-Y^r$	$-Y^q$	$Y^r + Y^q + Y^r + Y^7$			$-Y^r$	

b) Seventh row of Y^α

FIG. 7. Establishment of one row of $\mathbf{Y}^\alpha$.

Inverse Impedance Matrix **Z**

Altogether four such **Y** matrices are established. (If some or all of the matrices are identical, only one of them need be established.) In the present example $\mathbf{Y}^\gamma$ is a 15×15 matrix, while $\mathbf{Y}^\alpha$, $\mathbf{Y}^\beta$ and $\mathbf{Y}^\delta$ are 16×16 matrices.

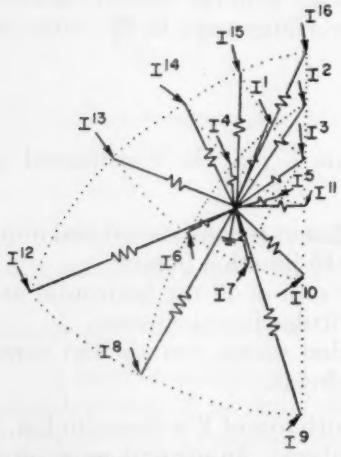


FIG. 8. Equivalent circuit of $\mathbf{Z}_\alpha$.

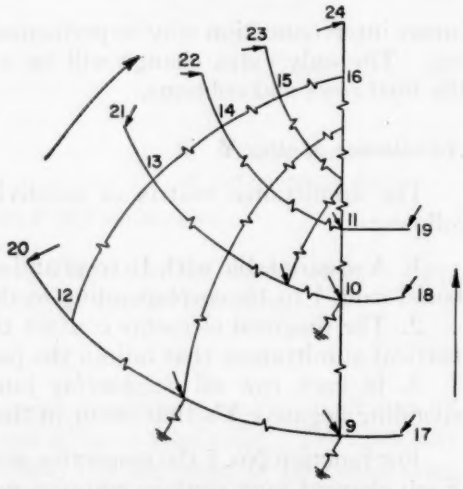


FIG. 9. Adding the boundary points of entry.

The inverse of each of these matrices is established in succession on the computer, forming $\mathbf{Z}_\alpha$ to $\mathbf{Z}_\delta$. The *schematic* equivalent circuit, representing $\mathbf{Z}_\alpha$ is shown in Fig. 8. The ground point is arbitrarily

assumed at any place, while the point of entry of each coil is located at its own particular junction. Each coil is mutually coupled. (The coupling arrows are not shown.)

Addition of Tie-Lines

Since subdivision α contains eight additional boundary points of entry (Fig. 9), $\mathbf{Z}_\alpha$ *should contain 24 rows and columns and not 16.* However, it will be shown now, that the missing 8 rows and columns can be built out of the rest in two ways (reference 4).

1. A rectangular transformation matrix $\mathbf{C}$ is established with 16 rows and 24 columns and the new $\mathbf{Z}_\alpha'$ is established by the formula $\mathbf{C}_t\mathbf{Z}_\alpha\mathbf{C}$.

This product is equivalent to interchanging and repeating some of the rows and columns of the original $\mathbf{Z}_\alpha$. *No additions or multiplications are needed to find the augmented* $\mathbf{Z}_\alpha'$. If the available computer is not equipped to do such permutations, then some actual matrix multiplications may be performed, as will be shown presently.

2. Several copies are printed of the inverse matrix $\mathbf{Z}_\alpha$. Certain rows or columns are removed by scissors and sometimes pasted together to form new matrices.

Since *in the present example* each boundary junction-point has only one boundary coil, *the added rows and columns are found by merely partitioning the original matrix* $\mathbf{Z}_\alpha$. Neither of the above two procedures is needed.

However, to make this paper useful for the calculation of more complex networks than that of Fig. 1, each of the above two possible steps will be described in greater detail presently.

Numbering the Boundary Points of Entry

The junction numbers 1–16 are extended to 24 by adding the 8 boundary coils, as shown in Fig. 9. For the present *the boundary coils contain zero impedances* (to be called "pigtails"); nevertheless, each of them contains an extra impressed current as shown.

The points of entry of the boundary coils are numbered beginning with 17 *in the direction of the arrows* and in the same order as the arrows were previously considered. (The reader not interested in more complex problems may skip the next three Sections.)

Transformation Matrix $\mathbf{C}$

1. If it is intended to use a digital computer to establish the added rows and columns, then the matrix of transformation $\mathbf{C}$ (representing

a relation between the 16 "old" $\mathbf{I}$ and the 24 "new" $\mathbf{I} + \mathbf{i}$) assumes the form

$$\mathbf{C} = \begin{array}{c} \\ 1\text{–}8 \\ 9\text{–}16 \end{array} \begin{array}{c} \begin{array}{cc} 1\text{–}8 & 9\text{–}16 \end{array} \quad 17\text{–}24 \\ \left[\begin{array}{cc|c} 1 & & \\ & 1 & \mathbf{C}_\alpha \end{array}\right] \end{array}. \tag{1}$$

Because of the radial arrangement of the subdivisions, *in the present example* $\mathbf{C}_\alpha$ *happens to be a unit matrix.* If however two or more pigtails were attached to the same junction point, then $\mathbf{C}_\alpha$ would differ from the unit matrix. *The formulae to be developed now will assume that* $\mathbf{C}_\alpha$ *differs from the unit matrix.*

2. If it is intended to use scissors and scotch-tape to form the augmented part of the $\mathbf{Z}_\alpha'$ matrix, then the 17–24 boundary axes of $\mathbf{C}$ should be split along the two neighboring subdivisions, as suggested by Fig. 9, so that $\mathbf{C}$ assumes the following partitioned form:

$$\mathbf{C} = \begin{array}{c} \\ 1\text{–}8 \\ 9\text{–}11 \\ 12\text{–}16 \end{array} \begin{array}{c} \begin{array}{cccc} 1\text{–}8 & 9\text{–}11 & 12\text{–}16 & \quad 17\text{–}19 \quad 20\text{–}24 \end{array} \\ \left[\begin{array}{ccc|c|c} 1 & & & & \\ & 1 & & \mathbf{C}_1 & \\ & & 1 & & \mathbf{C}_2 \end{array}\right] \end{array}. \tag{2}$$

In the present simple example both $\mathbf{C}_1$ and $\mathbf{C}_2$ are unit matrices.

Steps to Augment Inverse Matrix

The augmented matrix of subdivision α is found by

$$\mathbf{Z}_\alpha' = \mathbf{C}_t \mathbf{Z}_\alpha \mathbf{C} \tag{3}$$

containing 24 rows and columns. Two possibilities will now be discussed.

If a digital computer is used (when $\mathbf{C}_\alpha$ is not unity) then $\mathbf{Z}_\alpha$ is partitioned between the non-boundary axes (1–8) and boundary axes (9–16) as

$$\mathbf{Z}_\alpha = \begin{array}{c} \\ 1\text{–}8 \\ 9\text{–}16 \end{array} \begin{array}{c} \begin{array}{cc} 1\text{–}8 & 9\text{–}16 \end{array} \\ \left[\begin{array}{c|c} \mathbf{Z}_p & \mathbf{Z}_q \\ \mathbf{Z}_r & \mathbf{Z}_t \end{array}\right] \end{array} = [\mathbf{Z}_P \mathbf{Z}_Q] \tag{4}$$

and

$$\begin{array}{c} \\ 1\text{–}8 \\ 9\text{–}16 \end{array} \begin{array}{c} \begin{array}{cc} 1\text{–}8 & 9\text{–}16 \end{array} \\ \left[\begin{array}{cc} \mathbf{Z}_p & \mathbf{Z}_q \\ \hline \mathbf{Z}_r & \mathbf{Z}_t \end{array}\right] \end{array} = \begin{bmatrix} \mathbf{Z}_R \\ \mathbf{Z}_T \end{bmatrix}. \tag{5}$$

The augmented matrix assumes the form (using $\mathbf{C}$ of Eq. 1)

$$\mathbf{Z}_\alpha' = \begin{array}{c} \\ 1\text{–}8 \\ 9\text{–}16 \\ 17\text{–}24 \end{array} \begin{array}{c} \begin{array}{cc} 1\text{–}8 & 9\text{–}16 \end{array} \quad 17\text{–}24 \\ \left[\begin{array}{cc|c} \mathbf{Z}_p & \mathbf{Z}_q & \mathbf{Z}_q\mathbf{C}_\alpha \\ \mathbf{Z}_r & \mathbf{Z}_t & \mathbf{Z}_t\mathbf{C}_\alpha \\ \hline \mathbf{C}_{\alpha t}\mathbf{Z}_r & \mathbf{C}_{\alpha t}\mathbf{Z}_t & \mathbf{C}_{\alpha t}\mathbf{Z}_t\mathbf{C}_\alpha \end{array}\right] \end{array} \tag{6}$$

$$= \begin{array}{c} \\ 1\text{–}16 \\ 17\text{–}24 \end{array} \begin{array}{c} \begin{array}{cc} 1\text{–}16 & \end{array} \quad 17\text{–}24 \\ \left[\begin{array}{c|c} \mathbf{Z}_\alpha & \mathbf{Z}_Q\mathbf{C}_\alpha \\ \hline \mathbf{C}_{\alpha t}\mathbf{Z}_T & \mathbf{C}_{\alpha t}\mathbf{Z}_t\mathbf{C}_\alpha \end{array}\right] \end{array}. \tag{7}$$

The original inverse matrix $\mathbf{Z}_\alpha$ may thus be augmented by performing the matrix multiplications indicated in the last row and column. (If $\mathbf{Z}_\alpha'$ is symmetrical, only the last column need be calculated.)

Scissors and Scotch-Tape

If it is intended to paste together the additional rows and columns from the original inverse (without performing any matrix multiplications), then the boundary axes of the latter (9–16) are further partitioned along the axes of the neighboring subdivisions as

$$\mathbf{Z}_\alpha = \begin{array}{c} \\ 1\text{–}8 \\ 9\text{–}11 \\ 12\text{–}16 \end{array} \begin{array}{c} \begin{array}{ccc} 1\text{–}8 & 9\text{–}11 & 12\text{–}16 \end{array} \\ \left[\begin{array}{c|c|c} \mathbf{Z}_a & \mathbf{Z}_b & \mathbf{Z}_c \\ \mathbf{Z}_h & \mathbf{Z}_d & \mathbf{Z}_f \\ \mathbf{Z}_i & \mathbf{Z}_j & \mathbf{Z}_g \end{array}\right] \end{array} = \begin{array}{c} \\ 1\text{–}16 \end{array} \begin{array}{c} \begin{array}{ccc} 1\text{–}8 & 9\text{–}11 & 12\text{–}16 \end{array} \\ \left[\begin{array}{ccc} \mathbf{Z}_A & \mathbf{Z}_B & \mathbf{Z}_D \end{array}\right] \end{array} \tag{8}$$

and

$$\mathbf{Z}_\alpha = \begin{array}{c} \\ 1\text{–}8 \\ 9\text{–}11 \\ 12\text{–}16 \end{array} \begin{array}{c} \begin{array}{ccc} 1\text{–}8 & 9\text{–}11 & 12\text{–}16 \end{array} \\ \left[\begin{array}{ccc} \mathbf{Z}_a & \mathbf{Z}_b & \mathbf{Z}_c \\ \hline \mathbf{Z}_h & \mathbf{Z}_d & \mathbf{Z}_f \\ \hline \mathbf{Z}_i & \mathbf{Z}_j & \mathbf{Z}_g \end{array}\right] \end{array} = \begin{bmatrix} \mathbf{Z}_E \\ \mathbf{Z}_F \\ \mathbf{Z}_G \end{bmatrix}. \tag{9}$$

The matrix operations that have to be performed by pasting together various rows and columns, are shown in the last two columns of the augmented matrix

$$\mathbf{Z}_\alpha' = \begin{array}{c} \\ 1\text{–}16 \\ 17\text{–}19 \\ 20\text{–}24 \end{array} \begin{array}{c} \begin{array}{cc} 1\text{–}16 & 17\text{–}19 \end{array} \quad 20\text{–}24 \\ \left[\begin{array}{c|cc} \mathbf{Z}_\alpha & \mathbf{Z}_B\mathbf{C}_1 & \mathbf{Z}_D\mathbf{C}_2 \\ \hline \mathbf{C}_{1t}\mathbf{Z}_F & \mathbf{C}_{1t}\mathbf{Z}_d\mathbf{C}_1 & \mathbf{C}_{1t}\mathbf{Z}_f\mathbf{C}_2 \\ \mathbf{C}_{2t}\mathbf{Z}_G & \mathbf{C}_{2t}\mathbf{Z}_j\mathbf{C}_1 & \mathbf{C}_{2t}\mathbf{Z}_g\mathbf{C}_2 \end{array}\right] \end{array}. \tag{10}$$

The actual pasting procedure consists of the following steps:

(*a*) Post-multiplication with $\mathbf{C}$ consists of a rearrangement of *columns*.

(*b*) Pre-multiplication with $\mathbf{C}_t$ consists of a rearrangement of *rows*.

This latter step is performed on matrices (such as $\mathbf{Z}_d\mathbf{C}_1$ which is part of $\mathbf{Z}_B\mathbf{C}_1$ by Eq. 8) whose columns have already been rearranged.

Final Augmented Inverse

The augmented matrix (found either as Eq. 7 or as Eq. 10) will be represented from here on in the form

$$\mathbf{Z}_\alpha' = \begin{array}{c} \\ 1\text{–}16 \\ 17\text{–}19 \\ 20\text{–}24 \end{array} \left[\begin{array}{c|cc} 1\text{–}16 & 17\text{–}19 & 20\text{–}24 \\ \hline \mathbf{Z}_\alpha & \mathbf{A}_\alpha & \mathbf{B}_\alpha \\ \hline \mathbf{G}_\alpha & \mathbf{C}_\alpha & \mathbf{D}_\alpha \\ \mathbf{H}_\alpha & \mathbf{F}_\alpha & \mathbf{E}_\alpha \end{array}\right]. \tag{11}$$

It should be especially noted that the impedance Z of the boundary coils are not yet included.

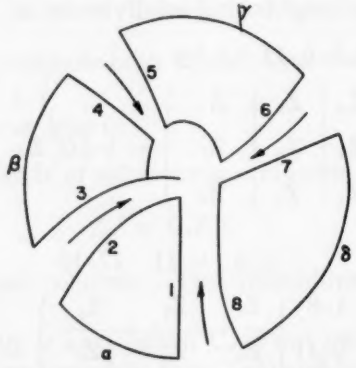


FIG. 10. Numbering the sets of pigtails.

Since in the present simple example both $\mathbf{C}_1$ and $\mathbf{C}_2$ are the unit matrix, the augmented matrix $\mathbf{Z}_\alpha'$ is found by simply removing $\mathbf{C}_1$ and $\mathbf{C}_2$ in Eq. 10.

$$\mathbf{Z}_\alpha' = \begin{array}{c} \\ 1\text{–}16 \\ 17\text{–}19 \\ 20\text{–}24 \end{array} \left[\begin{array}{c|cc} 1\text{–}16 & 17\text{–}19 & 20\text{–}24 \\ \hline \mathbf{Z}_\alpha & \mathbf{Z}_B & \mathbf{Z}_D \\ \hline \mathbf{Z}_F & \mathbf{Z}_d & \mathbf{Z}_f \\ \mathbf{Z}_G & \mathbf{Z}_j & \mathbf{Z}_g \end{array}\right]. \tag{12}$$

The addition of the two bordering rows and columns has been thus accomplished by simply partitioning the original matrix $\mathbf{Z}_\alpha$ as shown in Eqs. 8 and 9.

If $\mathbf{Z}_\alpha$ is symmetrical, the augmented matrix (Eq. 11) assumes the form

$$\mathbf{Z}_\alpha' = \begin{array}{c} \\ 1\text{–}16 \\ 17\text{–}19 \\ 20\text{–}24 \end{array} \begin{array}{c} \begin{array}{ccc} 1\text{–}16 & 17\text{–}19 & 20\text{–}24 \end{array} \\ \left[\begin{array}{c|cc} \mathbf{Z}_\alpha & \mathbf{A}_\alpha & \mathbf{B}_\alpha \\ \hline \mathbf{A}_{\alpha t} & \mathbf{C}_\alpha & \mathbf{D}_\alpha \\ \mathbf{B}_{\alpha t} & \mathbf{D}_{\alpha t} & \mathbf{E}_\alpha \end{array}\right] \end{array}. \tag{13}$$

Primitive System of the Subdivisions

Similar steps are repeated for each of the other three matrices $\mathbf{Z}_\beta$, $\mathbf{Z}_\gamma$, $\mathbf{Z}_\delta$.

The resultant matrix $\mathbf{Z}$ represents the "primitive" system of Fig. 5*a* existing *before* interconnection. Each set of boundary points (pigtails) is numbered, as shown in Fig. 10. On the two sides of each arrow appears such a number.

The resultant asymmetrical impedance matrix of the primitive system (without the boundary-coil impedances) is

$$\mathbf{Z} = \begin{array}{c} \\ \alpha \\ \beta \\ \gamma \\ \delta \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} \alpha & \beta & \gamma & \delta & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \left[\begin{array}{cccc|cccccccc} \mathbf{Z}_\alpha & & & & \mathbf{A}_\alpha & \mathbf{B}_\alpha & & & & & & \\ & \mathbf{Z}_\beta & & & & & \mathbf{A}_\beta & \mathbf{B}_\beta & & & & \\ & & \mathbf{Z}_\gamma & & & & & & \mathbf{A}_\gamma & \mathbf{B}_\gamma & & \\ & & & \mathbf{Z}_\delta & & & & & & & \mathbf{A}_\delta & \mathbf{B}_\delta \\ \hline \mathbf{G}_\alpha & & & & \mathbf{C}_\alpha & \mathbf{D}_\alpha & & & & & & \\ \mathbf{H}_\alpha & & & & \mathbf{F}_\alpha & \mathbf{E}_\alpha & & & & & & \\ & \mathbf{G}_\beta & & & & & \mathbf{C}_\beta & \mathbf{D}_\beta & & & & \\ & \mathbf{H}_\beta & & & & & \mathbf{F}_\beta & \mathbf{E}_\beta & & & & \\ & & \mathbf{G}_\gamma & & & & & & \mathbf{C}_\gamma & \mathbf{D}_\gamma & & \\ & & \mathbf{H}_\gamma & & & & & & \mathbf{F}_\gamma & \mathbf{E}_\gamma & & \\ & & & \mathbf{G}_\delta & & & & & & & \mathbf{C}_\delta & \mathbf{D}_\delta \\ & & & \mathbf{H}_\delta & & & & & & & \mathbf{F}_\delta & \mathbf{E}_\delta \end{array}\right] \end{array}. \tag{14}$$

$$\mathbf{Z} = \begin{array}{c} \\ \alpha\text{–}\delta \\ 1\text{–}8 \end{array} \begin{array}{c} \begin{array}{cc} \alpha\text{–}\delta & 1\text{–}8 \end{array} \\ \begin{bmatrix} \mathbf{Z}_1 & \mathbf{Z}_2 \\ \mathbf{Z}_3 & \mathbf{Z}_4 \end{bmatrix} \end{array}. \tag{15}$$

When $\mathbf{Z}$ is symmetrical $\mathbf{Z}_1$ and $\mathbf{Z}_4$ are symmetrical and $\mathbf{Z}_3$ is the transpose of $\mathbf{Z}_2$. *The resultant matrix (14) need not be established.* All its component matrices are, however, available for further manipulation.

The "tensor" equivalent circuit of the primitive system is shown in Fig. 11*a*. All impedances of a subdivision that are not part of the boundary, are represented by one "tensor" impedance drawn in heavy lines. Also each set of boundary coils is combined into one tensor coil.

Connection Matrix

The eight sets of pigtails existing before interconnection become four sets after interconnection. From Fig. 11*b* the matrix of transformation of the boundary coils, which gives the "old" boundary currents in terms of the "new," is

$$\mathbf{C}_b = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 2' & 4' & 6' & 8' \end{array} \\ \begin{bmatrix} & & & -1 \\ 1 & & & \\ -1 & & & \\ & 1 & & \\ & -1 & & \\ & & 1 & \\ & & -1 & \\ & & & 1 \end{bmatrix} \end{array}. \tag{16}$$

The connection matrix of the entire system of Fig. 11*b* is thus

$$\mathbf{C} = \begin{array}{c} \\ \alpha\text{–}\delta \\ 1\text{–}8 \end{array} \begin{array}{c} \begin{array}{cc} \alpha\text{–}\delta & 2'\text{–}8' \end{array} \\ \begin{bmatrix} 1 & \\ & \mathbf{C}_b \end{bmatrix} \end{array}. \tag{17}$$

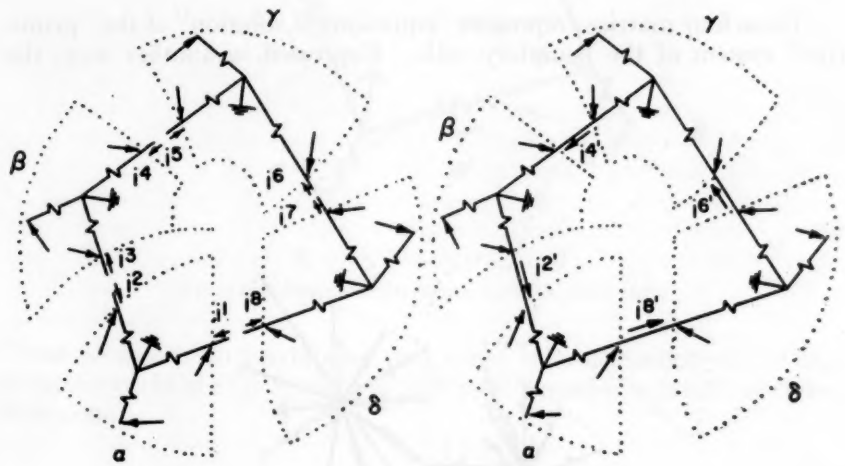
Interconnected System

The impedance matrix of the resultant system of Fig. 11*b* is by $\mathbf{C}_t\mathbf{Z}\mathbf{C}$

$$\mathbf{Z}' = \begin{array}{c} \\ \alpha\text{–}\delta \\ 2'\text{–}8' \end{array} \begin{array}{c} \begin{array}{cc} \alpha\text{–}\delta & 2'\text{–}8' \end{array} \\ \begin{bmatrix} \mathbf{Z}_1 & \mathbf{Z}_2\mathbf{C}_b \\ \mathbf{C}_{bt}\mathbf{Z}_3 & \mathbf{C}_{bt}\mathbf{Z}_4\mathbf{C}_b \end{bmatrix} \end{array} = \begin{bmatrix} \mathbf{Z}_1 & \mathbf{Z}_2' \\ \mathbf{Z}_3' & \mathbf{Z}_4' \end{bmatrix}. \tag{18}$$

Performing the indicated multiplications (on paper only) $\mathbf{Z}_1$ remains unchanged, while

$$\mathbf{Z}_2' = \begin{array}{c} \\ \alpha \\ \beta \\ \gamma \\ \delta \end{array} \begin{array}{c} \begin{array}{cccc} 2' & 4' & 6' & 8' \end{array} \\ \begin{bmatrix} \mathbf{B}_\alpha & & & -\mathbf{A}_\alpha \\ -\mathbf{A}_\beta & \mathbf{B}_\beta & & \\ & -\mathbf{A}_\gamma & \mathbf{B}_\gamma & \\ & & -\mathbf{A}_\delta & \mathbf{B}_\delta \end{bmatrix} \end{array}. \tag{19}$$



a) Primitive system (with pigtails)

b) Interconnected system (no boundary coils)

FIG. 11. Tensor networks.

$\mathbf{Z}_3'$ is the transpose of $\mathbf{Z}_2'$. Finally

$$\mathbf{Z}_4' = \begin{array}{c} \\ 2' \\ 4' \\ 6' \\ 8' \end{array} \begin{array}{c} \begin{array}{cccc} 2' & 4' & 6' & 8 \end{array} \\ \begin{bmatrix} \mathbf{E}_\alpha + \mathbf{C}_\beta & -\mathbf{D}_\beta & & -\mathbf{F}_\alpha \\ -\mathbf{F}_\beta & \mathbf{E}_\beta + \mathbf{C}_\gamma & -\mathbf{D}_\gamma & \\ & -\mathbf{F}_\gamma & \mathbf{E}_\gamma + \mathbf{C}_\delta & -\mathbf{D}_\delta \\ -\mathbf{D}_\alpha & & -\mathbf{F}_\delta & \mathbf{E}_\delta + \mathbf{C}_\alpha \end{bmatrix} \end{array}. \tag{20}$$

The effect of the interconnection $\mathbf{C}_i\mathbf{Z}\mathbf{C}$ of the four subdivisions is merely to *add* a set of two matrices $\mathbf{E}$ and $\mathbf{C}$ along the diagonal of $\mathbf{Z}_4'$.

Solving the Removed Boundaries

The impedances $\mathbf{Z}$ of each cut boundary coil (along the arrows of Fig. 10) can be arranged along the diagonals of a matrix. Four such diagonal matrices may be established. For instance, the three impedances along $\mathbf{Z}_8'$ of Fig. 12 and Fig. 11*b* are

$$\mathbf{Z}_8 = \begin{bmatrix} Z_p & & \\ & Z_q & \\ & & Z_r \end{bmatrix}. \tag{21}$$

These four matrices represent "equations of solution" of the "primitive" system of the boundary coils. Expressed in another way, the

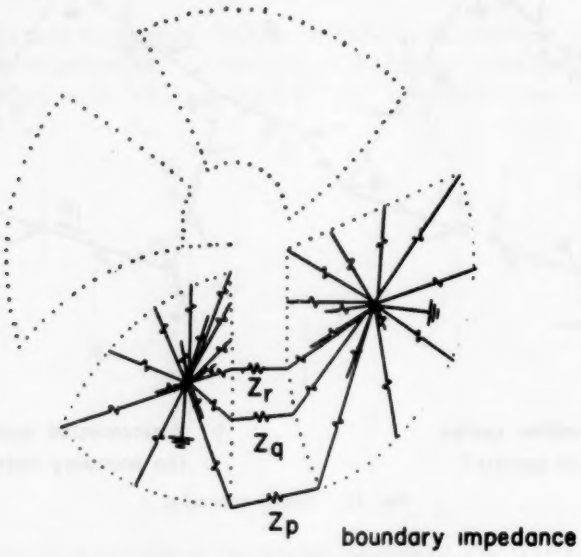


FIG. 12. Adding boundary impedances.

equations $\mathbf{I}^b = \mathbf{Y}^b\mathbf{E}_b$ of the removed boundary system of Fig. 5*b* may be solved outright by simple inspection as $\mathbf{E}_b = \mathbf{Z}_b\mathbf{I}^b$.

Adding the Boundary System

These impedance matrices are simply added to the diagonal elements of $\mathbf{Z}_4'$ in the proper order as follows:

$$\begin{array}{l|l} \mathbf{Z}_2' = \mathbf{E}_\alpha + \mathbf{C}_\beta + \mathbf{Z}_2 & \mathbf{Z}_6' = \mathbf{E}_\gamma + \mathbf{C}_\delta + \mathbf{Z}_6 \\ \mathbf{Z}_4' = \mathbf{E}_\beta + \mathbf{C}_\gamma + \mathbf{Z}_4 & \mathbf{Z}_8' = \mathbf{E}_\delta + \mathbf{C}_\alpha + \mathbf{Z}_8. \end{array} \tag{22}$$

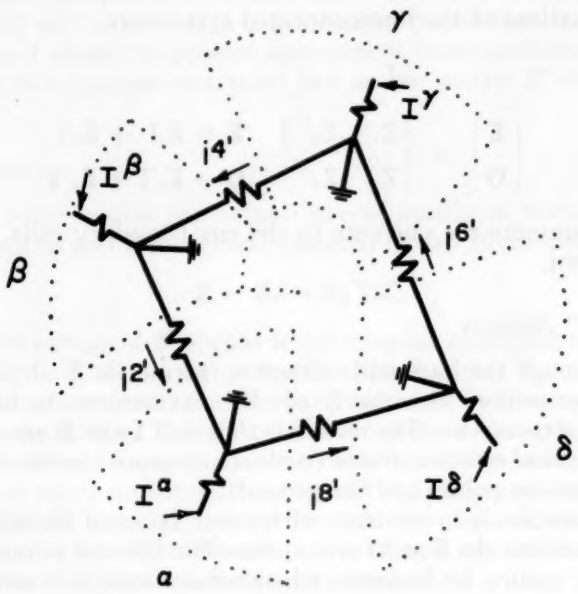


FIG. 13. Interconnected system with boundary coils.

These additions do correspond to a series interconnection of the two mesh networks of Fig. 5*b* and Fig 11*b* with the aid of a matrix of transformation

$$\mathbf{C} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{23}$$

The resultant $\mathbf{Z}$ of the interconnected system, including the cut boundary coils, is shown in Eq. 24.

$$\mathbf{Z}' = \begin{array}{c|cccc|cccc} & \alpha & \beta & \gamma & \delta & 2' & 4' & 6' & 8' \\ \alpha & \mathbf{Z}_\alpha & & & & \mathbf{B}_\alpha & & & -\mathbf{A}_\alpha \\ \beta & & \mathbf{Z}_\beta & & & -\mathbf{A}_\beta & \mathbf{B}_\beta & & \\ \gamma & & & \mathbf{Z}_\gamma & & & -\mathbf{A}_\gamma & \mathbf{B}_\gamma & \\ \delta & & & & \mathbf{Z}_\delta & & & -\mathbf{A}_\delta & \mathbf{B}_\delta \\ \hline 2' & \mathbf{H}_\alpha & -\mathbf{G}_\beta & & & \mathbf{Z}_2' & -\mathbf{D}_\beta & & -\mathbf{F}_\alpha \\ 4' & & \mathbf{H}_\beta & -\mathbf{G}_\gamma & & -\mathbf{F}_\beta & \mathbf{Z}_4' & -\mathbf{D}_\gamma & \\ 6' & & & \mathbf{H}_\gamma & -\mathbf{G}_\delta & & -\mathbf{F}_\gamma & \mathbf{Z}_6' & -\mathbf{D}_\delta \\ 8' & -\mathbf{G}_\alpha & & & \mathbf{H}_\delta & -\mathbf{D}_\alpha & & -\mathbf{F}_\delta & \mathbf{Z}_8' \end{array}. \tag{24}$$

The equations of the interconnected system are

$$\begin{matrix} & \begin{matrix} \mathbf{I} & \mathbf{i} \end{matrix} & \\ \begin{bmatrix} \mathbf{E} \\ \mathbf{O} \end{bmatrix} = & \begin{bmatrix} \mathbf{Z}_1 & \mathbf{Z}_2' \\ \mathbf{Z}_3' & \mathbf{Z}_4'' \end{bmatrix} & \begin{matrix} \mathbf{E} = \mathbf{Z}_1\mathbf{I} + \mathbf{Z}_2'\mathbf{i} \\ \mathbf{O} = \mathbf{Z}_3'\mathbf{I} + \mathbf{Z}_4''\mathbf{i} \end{matrix} \end{matrix} \tag{25}$$

where **i** represents the currents in the cut boundary coils. They are still unknown.

"Factorized" Solution

Even though the **i** are still unknown, the matrix $\mathbf{Z}'$ already may be used—in connection with the $\mathbf{Z}'$ of other structures—to build up the $\mathbf{Z}'$ of super-structures. The reason is that *both* **I** *and* **E** *are expressed in the same physical reference frame* (both are measured between the actual *physical* junction points and the ground).

This situation is in contrast with other types of *factorized form* of solutions in which the **E** and **I** are expressed in different reference frames. Such a case occurs, for instance, whenever a matrix is inverted by conventional partitioning. At every intermediate step the **E** and **I** are expressed in different reference frames and the intermediate $\mathbf{Z}'$ cannot be used for the construction of superstructures. For such purposes it is necessary to establish the actual inverse matrix with all its elements different from zero.

"Factorized" Inverse Table

The matrix of $\mathbf{Z}_4''$ has 16 rows and columns (as many as there are cut coils). Its inverse may be found on the computer in one step as $\mathbf{Y}^4$.

If **i** is eliminated from the second equation, the first equation reduces to

$$\begin{aligned} \mathbf{E} &= (\mathbf{Z}_1 - \mathbf{Z}_2'\mathbf{Y}^4\mathbf{Z}_3')\mathbf{I}. \\ \mathbf{E} &= \mathbf{Z}\mathbf{I} \end{aligned} \tag{26}$$

The indicated operations should not be performed, but the four matrices should be left arranged in a table with 79 rows and columns. (This table is not a matrix.)

$$\mathbf{Z}_T = \begin{matrix} & \begin{matrix} 63 & 16 \end{matrix} \\ \begin{matrix} 63 \\ 16 \end{matrix} & \begin{bmatrix} \mathbf{Z}_1 & \mathbf{Z}_2' \\ \mathbf{Z}_3' & \mathbf{Y}^4 \end{bmatrix} \end{matrix}. \tag{27}$$

It is called "factorized" inverse table, as the four matrices are factors of the actual inverse matrix **Z** by Eq. 26.

For many purposes the above table takes the place of an actual inverse. It will be shown in a subsequent paper that the same table may be used when the present structure is interconnected with other structures into a super-structure, just as the matrix $\mathbf{Z}'$ of Eq. 24 can be used.

Actual Inverse

If for some reason the actual inverse matrix is wanted, then the multiplications and subtraction indicated in Eq. 26 are performed as

$$\mathbf{Z} = \mathbf{Z}_1 - \mathbf{Z}_2'\mathbf{Y}^4\mathbf{Z}_3'. \tag{28}$$

A disadvantage of $\mathbf{Z}$ is, that it contains far more non-zero elements than do all four of the factor-matrices combined. In fact if n is the number of subdivisions, then $\mathbf{Z}$ contains about $\sqrt{n}$ times as many non-zero elements as does Table 27. (In the present example, about twice as many.) Hence for any assumed value of $\mathbf{i}$, it is necessary to perform $\sqrt{n}$ times as many multiplications to find the corresponding $\mathbf{E}$.

It should be especially noted that the reference frame of $\mathbf{E}$ is the same as that of $\mathbf{I}$ in $\mathbf{Z}$, in $\mathbf{Z}_3^2$ and in $\mathbf{Z}'$. Hence each of them is equally valid for the construction of superstructures.

Calculation of $\mathbf{E}$

For any given set of $\mathbf{I}$ (a vector with 63 elements) the corresponding $\mathbf{E}$ vector is calculated in the following order:

1. The voltage vector producing the circulating currents is

$$\mathbf{e} = \mathbf{Z}_3'\mathbf{I}. \tag{29}$$

2. The circulating current-vector is

$$\mathbf{i} = -\mathbf{Y}^4\mathbf{e}. \tag{30}$$

3. The differences of potential $\mathbf{E}$ between each junction-point and the ground is

$$\mathbf{E} = \mathbf{Z}_1\mathbf{I} + \mathbf{Z}_2'\mathbf{i}. \tag{31}$$

The number of multiplications in finding $\mathbf{E}$ is only about $1/\sqrt{n} = 1/\sqrt{4}$ or one-half of the multiplications, if the inverse $\mathbf{Z}$ of Eq. 28 had actually been calculated and used.

Total Machine and Clerical Work

In finding the "factorized" or "orthogonal" inverse of the 63×63 matrix of Eq. 24 on the computer, the following operations are to be performed on the computer:

1. The inversion of the four matrices $\mathbf{Z}_\alpha$–$\mathbf{Z}_\delta$.
2. The inversion of $\mathbf{Z}_4''$.
3. The eight matrix additions indicated in Eq. 22.

The total operation should take about 6 hours on the CPC. (Ignoring input and output time, punching time, etc. The inversion of a 16×16 matrix is assumed to take about 70 minutes.)

The clerical work to perform the interconnection $\mathbf{C}_t\mathbf{ZC}$ includes only the partitioning of the four inverse matrices $\mathbf{Z}_\alpha$–$\mathbf{Z}_\delta$ into the component matrices **A**–**E** of Eq. 14. No rearrangement of rows and columns is needed for the present example.

Improved Accuracy of Inversion

It should be noted that each element of the original **Y** matrix of the entire system takes part in a maximum of two successive inversion processes:

1. The 16×16 **Y** matrix of each isolated subdivision is inverted once.
2. From the results of the above inversions a boundary 16×16 $\mathbf{Z}_4$ matrix is formed which is inverted again.

Hence each element of $\mathbf{Z}_4$ is subjected to a set of manipulations equivalent roughly to the inversion of a 32×32 matrix. However the elements of $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$ matrices have been subjected altogether to only a 16×16 inversion.

It may be said that the accuracy of the factorized inverse of the original 63×63 matrix is equal or better than the accuracy of some equivalent 32×32 inverse matrix. This ratio of improvement in the accuracy of an inverse calculated by the method of tearing—compared with the accuracy of a conventionally inverted matrix—increases with the number of subdivisions. Moreover this ratio of improvement in accuracy is more or less independent of the nature of the eigen-values and their over-all dispersion.

If necessary, the inverse of the boundary $\mathbf{Z}_4$ may be improved by standard methods, thus improving simultaneously the over-all inverse.

NUMERICAL EXAMPLE

Solution of Each Subdivision

Let the system of Fig. 14 with equations $\mathbf{I} = \mathbf{YE}$ be given, containing eight junction-pairs.

The system is torn apart into two identical parts, as shown in Fig. 15*a*. The two boundary coils, across which the cut is made, are temporarily laid aside as shown in Fig. 15*b*.

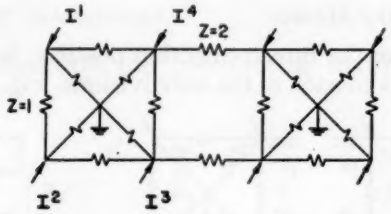


FIG. 14. Given physical system (given $\mathbf{I} = \mathbf{YE}$).

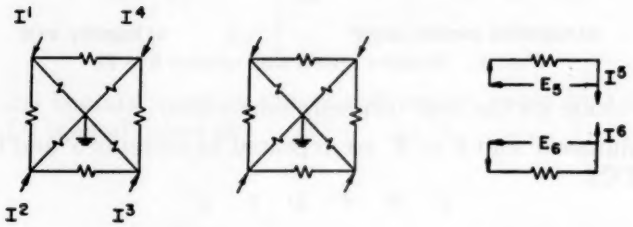


FIG. 15. Tearing apart the system.

The $\mathbf{I} = \mathbf{YE}$ equations of each subdivision (without the boundary coils) as

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} I^1 \\ I^2 \\ I^3 \\ I^4 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 3 & -1 & & -1 \\ -1 & 3 & -1 & \\ & -1 & 3 & -1 \\ -1 & & -1 & 3 \end{bmatrix} \end{array} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix}. \tag{32}$$

The inverse of each $\mathbf{Y}$ matrix is

$$\mathbf{Z} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 7 & 3 & 2 & 3 \\ 3 & 7 & 3 & 2 \\ 2 & 3 & 7 & 3 \\ 3 & 2 & 3 & 7 \end{bmatrix} \end{array} \frac{1}{15} \tag{33}$$

representing the equation of solution of each subdivision as $\mathbf{E} = \mathbf{ZI}$.

Addition of Boundary Meshes

In order to make the interconnection possible, let two extra meshes be added at the cuts to each of the subdivisions, Fig. 16. The following

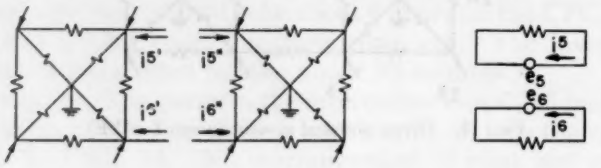


FIG. 16. Primitive system after solution $\mathbf{E} = \mathbf{ZI}$.

steps are made for the first (primed) subdivision:

1. Columns 3′ and 4′ of $\mathbf{Z}'$ are repeated as columns 5′ and 6′ (representing $\mathbf{Z}'\mathbf{C}$).

$$\begin{array}{c} \\ 1' \\ 2' \\ 3' \\ 4' \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} 1' & 2' & 3' & 4' & 5' & 6' \end{array} \\ \left[\begin{array}{cccc|cc} 7 & 3 & 2 & 3 & 2 & 3 \\ 3 & 7 & 3 & 2 & 3 & 2 \\ 2 & 3 & 7 & 3 & 7 & 3 \\ 3 & 2 & 3 & 7 & 3 & 7 \end{array}\right] \end{array} \frac{1}{15}. \tag{34}$$

2. Rows 3′ and 4′ of the augmented matrix are repeated as rows 5′ and 6′ (representing $\mathbf{C}_t\mathbf{Z}'\mathbf{C}$).

$$\mathbf{Z}' = \begin{array}{c} \\ 1' \\ 2' \\ 3' \\ 4' \\ 5' \\ 6' \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} 1' & 2' & 3' & 4' & 5' & 6' \end{array} \\ \left[\begin{array}{cccc|cc} 7 & 3 & 2 & 3 & 2 & 3 \\ 3 & 7 & 3 & 2 & 3 & 2 \\ 2 & 3 & 7 & 3 & 7 & 3 \\ 3 & 2 & 3 & 7 & 3 & 7 \\ \hline 2 & 3 & 7 & 3 & 7 & 3 \\ 3 & 2 & 3 & 7 & 3 & 7 \end{array}\right] \end{array} \frac{1}{15} = \begin{array}{c} \\ j' \\ m' \end{array}\!\!\begin{array}{c} \begin{array}{cc} j' & m' \end{array} \\ \left[\begin{array}{cc} \mathbf{Z}_1' & \mathbf{Z}_2' \\ \mathbf{Z}_3' & \mathbf{Z}_4' \end{array}\right] \end{array}. \tag{35}$$

The $\mathbf{Z}'$ matrix is partitioned between the junction-pairs $\mathbf{j}$ and boundary meshes $\mathbf{m}$.

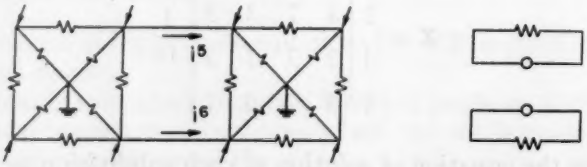


FIG. 17. Interconnected system (boundary coils separate).

Primitive System of Subdivisions

Before interconnection the resultant **Z** matrix of the two subdivisions is

$$\mathbf{Z} = \begin{array}{c} \\ j' \\ j'' \\ m' \\ m'' \end{array} \begin{array}{c} \begin{array}{cccc} j' & j'' & m' & m'' \end{array} \\ \left[\begin{array}{cc|cc} \mathbf{Z}_1' & & \mathbf{Z}_2' & \\ & \mathbf{Z}_1'' & & \mathbf{Z}_2'' \\ \hline \mathbf{Z}_3' & & \mathbf{Z}_4' & \\ & \mathbf{Z}_3'' & & \mathbf{Z}_4'' \end{array}\right] \end{array} = \begin{array}{c} \\ j \\ m \end{array} \begin{array}{c} \begin{array}{cc} j & m \end{array} \\ \left[\begin{array}{cc} \mathbf{Z}_a & \mathbf{Z}_b \\ \mathbf{Z}_c & \mathbf{Z}_d \end{array}\right] \end{array}. \tag{36}$$

For the boundary coils a diagonal **Z** matrix (equations of solution $\mathbf{E}_b = \mathbf{Z}_b\mathbf{I}^b$) is established as

$$\mathbf{Z}_b = \begin{array}{c} \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cc} 5 & 6 \end{array} \\ \left[\begin{array}{cc} 2 & \\ & 2 \end{array}\right] \end{array}. \tag{37}$$

Connection Tensor **C**

The interconnection of junction-pairs and meshes of the two subdivisions is performed by $\mathbf{I} = \mathbf{C}\mathbf{I}'$

$$\mathbf{C} = \begin{array}{c} \\ j \\ m \end{array} \begin{array}{c} \begin{array}{cc} j & m \end{array} \\ \left[\begin{array}{cc} 1 & \\ & \mathbf{C}_m \end{array}\right] \end{array} \tag{38}$$

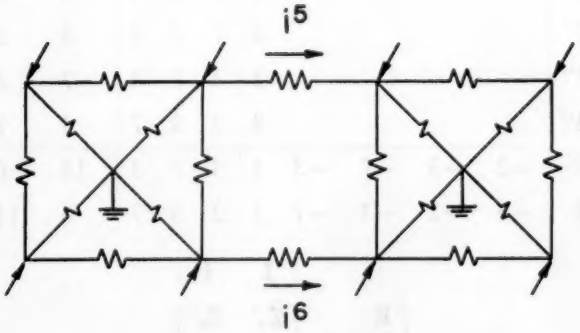


FIG. 18. Addition of boundary coils.

where the meshes are interconnected by

$$\mathbf{C}_m = \begin{array}{c} \\ 5' \\ 6' \\ 5'' \\ 6'' \end{array} \begin{array}{c} \begin{array}{cc} 5 & 6 \end{array} \\ \left[\begin{array}{cc} -1 & \\ & -1 \\ \hline 1 & \\ 1 & \end{array}\right] \end{array} = \begin{array}{c} \\ m' \\ m'' \end{array} \begin{array}{c} \\ \left[\begin{array}{c} -1 \\ 1 \end{array}\right] \end{array}. \tag{39}$$

The boundary coils are still left isolated.

Interconnected System

The $\mathbf{Z}'$ matrix of the interconnected system is by $\mathbf{C}_t\mathbf{Z}\mathbf{C}$

$$\begin{array}{c} \\ j' \\ j'' \\ m \end{array} \begin{array}{c} \begin{array}{cc|c} j' & j'' & m \\ \mathbf{I}' & \mathbf{I}'' & \mathbf{i} \end{array} \\ \left[\begin{array}{cc|c} \mathbf{Z}_1' & & -\mathbf{Z}_2' \\ & \mathbf{Z}_1'' & \mathbf{Z}_2'' \\ \hline -\mathbf{Z}_3' & \mathbf{Z}_3'' & \mathbf{Z}_4' + \mathbf{Z}_4'' \end{array}\right] \end{array} = \begin{array}{c} \\ \mathbf{E}' \\ \mathbf{E}'' \\ \mathbf{O} \end{array} \begin{array}{c} \\ \left[\begin{array}{c} \\ \\ \hline \\ \end{array}\right] \end{array} \tag{40}$$

$$\mathbf{Z}' = \begin{array}{c} \\ 1' \\ 2' \\ 3' \\ 4' \\ 1'' \\ 2'' \\ 3'' \\ 4'' \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccccc|cc} 1' & 2' & 3' & 4' & 1'' & 2'' & 3'' & 4'' & 5 & 6 \end{array} \\ \left[\begin{array}{cccccccc|cc} 7 & 3 & 2 & 3 & & & & & -2 & -3 \\ 3 & 7 & 3 & 2 & & & & & -3 & -2 \\ 2 & 3 & 7 & 3 & & & & & -7 & -3 \\ 3 & 2 & 3 & 7 & & & & & -3 & -7 \\ & & & & 7 & 3 & 2 & 3 & 2 & 3 \\ & & & & 3 & 7 & 3 & 2 & 3 & 2 \\ & & & & 2 & 3 & 7 & 3 & 7 & 3 \\ & & & & 3 & 2 & 3 & 7 & 3 & 7 \\ \hline -2 & -3 & -7 & -3 & 2 & 3 & 7 & 3 & 14 & 6 \\ -3 & -2 & -3 & -7 & 3 & 2 & 3 & 7 & 6 & 14 \end{array}\right] \end{array} \frac{1}{15} \tag{41}$$

$$\begin{array}{c} \\ \mathbf{E} \\ \mathbf{O} \end{array} \begin{array}{c} \begin{array}{cc} \mathbf{I} & \mathbf{i} \end{array} \\ \left[\begin{array}{cc} \mathbf{Z}_a' & \mathbf{Z}_b' \\ \mathbf{Z}_c' & \mathbf{Z}_d' \end{array}\right] \end{array}. \tag{42}$$

The currents **i** flowing in the boundary coils are unknown, while **I** is known.

Adding the Boundary Coils

The impedance matrix $\mathbf{Z}_b$ of the removed boundary coils (Eq. 37) is added to $\mathbf{Z}_d'$ as

$$\mathbf{Z}_d'' = \begin{bmatrix} 44 & 6 \\ 6 & 44 \end{bmatrix} \frac{1}{15}. \tag{43}$$

Factorized Inverse

The inverse of $\mathbf{Z}_d''$ is

$$\mathbf{Y}_d'' = \begin{bmatrix} 44/15 & -6/15 \\ -6/15 & 44/15 \end{bmatrix} \frac{9}{76}. \tag{44}$$

The final factorized inverse table is thus

$$\mathbf{Z}'' = \begin{bmatrix} \mathbf{Z}_a' & \mathbf{Z}_b' \\ \mathbf{Z}_c' & \mathbf{Y}_d'' \end{bmatrix}. \tag{45}$$

Three of the matrices are given in Eq. 41, the fourth in Eq. 44.

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Machine to Produce Transistors.—A machine that can automatically carry out a series of more than fifteen intricate steps in making experimental transistors—the solid pea-sized amplifiers that have most of the advantages of vacuum tubes—is now in an early stage of development at Bell Telephone Laboratories.

Here is what the unique machine, familiarly known as "Mr. Meticulous," will do in less than one minute:

Take a tiny bar of a special semiconducting material, such as germanium or silicon—a bar almost as thin as a human hair—examine it carefully for electrical characteristics and accept or reject it. If it accepts it, the machine fixes a fine gold wire to a critical point on the bar within an accuracy of 1/20,000 of an inch. Then, still within one minute, it connects this wire to one of the four wires leading out of the transistor, flips the bar end over end, repeats the entire operation with another wire on the opposite side, and finally runs a series of electrical tests on the completed transistor.

Transistors for research purposes must be of the highest precision and accuracy. When fashioned by human hands over any extended period of time, some transistors are produced which are substandard and useless for research purposes. This is the result of fatigue on the part of the technician. But, "Mr. Meticulous" never gets tired, never loses his precision or accuracy. His hand never shakes and his highly organized electronic "brain" rarely has mental lapses.

The machine, originated by R. L. Wallace of Bell Telephone Laboratories, where the transistor was invented, may some day be a pilot model for industrial machines to be used in assembly-line transistor manufacture. At this stage, however, "Mr. Meticulous" is primarily a laboratory device designed to aid research on so-called "junction" transistors.

The machine operates this way: An operator places an n-p-n "sandwich" of germanium, as short as a matchhead and only a little thicker than a human hair, into a clamp on the machine. The machine then presses a very thin strand of gold wire against the bar and the wire edges along the bar with minute steps of 1/20,000 of an inch; after each step the device takes a quick electrical look to see whether it has reached the thin (1/10,000 of an inch) central layer of germanium, to which it must bond the wire. As soon as the wire touches this layer, the machine starts measuring width until the wire reaches the far side of the layer. The machine then decides whether the bar is satisfactory; if it is not, it can be automatically rejected.

If the bar is acceptable, the machine retraces its steps to the other side of the central layer and counts forward a predetermined number of steps. At this point a shot of electric current is used to attach the wire to the germanium. The machine then attaches this wire to one of the four leads of the transistor itself. It then rotates the bar of germanium end over end and automatically goes through the same series of operations on the other side of the bar.

Finally the machine runs a series of electrical tests to determine whether the completed transistor has the proper operating characteristics for research purposes.

If the transistor fails any of the tests, it can be automatically rejected. If it passes, "Mr. Meticulous" puts it aside as finished business and goes on to the next bar of transistor material.

BEAMS OF METAL OR WOOD SUBJECTED TO MILLIONS OF REVERSALS OF COMPRESSION AND TENSION ($\sigma_c = -\sigma_t$) OR TO LONGTIME STEADY COMPRESSION LONGITUDINALLY*

BY

ARTHUR BUCHWALD LAMBERG [1]

Beams of metal or wood subjected to millions of reversals of compression and tension ($\sigma_c = -\sigma_t$), or to longtime steady compression longitudinally, play an important role not only in architecture but also in civil and mechanical engineering. However, there are still difficulties which impair the reliable easy design of beams in network constructions, of columns and poles, of piston rods and connecting rods in engines, of beams in airplane wings, etc.

One difficulty is the danger in using vague safety factors for the utilization of the known formulae which give the critical load,[2] but not the admissible stress per unit cross-section area of beams subjected to longtime compression longitudinally.

Professor L. von Tetmajer used the equation

$$\sigma_k = \beta_k \cdot \frac{\sigma_d}{\beta_d},$$

namely, admissible load = critical load $\times \frac{\text{"pure compression"}}{\text{modulus of crush}}$ (every force per square cross section unit).

When the ratio l/i of the beam is greater than $\pi \sqrt{\frac{E}{S_P}}$, then

$$\beta_k = \frac{\pi^2 E i^2}{l^2_{\text{flexible}}}$$

according to Euler's derivation.

When l/i is less than $\pi \sqrt{\frac{E}{S_P}}$, then

$$\beta_k = \left(1 - a \frac{l}{i}\right) \cdot \beta_d$$

* Extract of a treatise prepared and copyrighted in 1951 by the author, who has granted the JOURNAL permission to publish this copyrighted extract.

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[2] That is the load, which immediately, or very soon after it has been brought up, causes tirfling bending of the beam.

as determined by Tetmajer for iron, steel and wood.

In both cases the "pure compression" is a vague force so that the fraction $\frac{\sigma_d}{\beta_d}$ represents a vague safety-factor by which the critical load β_k has to be multiplied to give the admissible load.

In the expression $(1 - a\,l/i)$ the length *denotes a vague part of the whole length* of the beam and the coefficient a must be determined by longtime tests for every kind of metal or wood from which the actual beam is to be made.

Another difficulty is in the problem of how to utilize the endurance limit F_{-1}, as determined on test specimens in rotating beam machines or Haigh-machines, when actual beams with any desired ratio $l/i_{\min.}$ shall be designed. Is it logical to represent σ_d in Tetmajer's formula $\frac{\sigma_k}{\sigma_d} = \frac{\beta_k}{\beta_d}$ by F_{-1}, which depends on the ratio l/i of the test specimen and therefore is a vague limit too?

All these difficulties can be overcome by utilization of the formula

$$F_{-1} = S_P \times (1 - q)$$

as derived by the author for the computation of the endurance limit of beams of metal or wood with any ratio l/i subjected to millions of lateral, longitudinal or torsional vibrations.

The author explains in his treatise why the endurance limit of a beam of metal or wood subjected to a longtime steady load longitudinally must be equal to or may be only a little higher than the endurance limit of the same beam subjected to millions of reversals $(\sigma_c = -\sigma_t)$ of compression and tension longitudinally.

The formula $F_{-1} = S_P \times (1 - q)$ was published in five articles.[3] S_P denotes the proportional compression limit of the material, q the resilience of the beam.

From this fundamental formula, the author recently derived the formula

$$F_{-1} = S_P[1 - q_{\text{special}}] \tag{1}$$

for beams with the ratio $l/i = \pi\sqrt{\frac{E}{S_{P\text{ or }E}}}$;

the formula

$$F_{-1} = S_P[1 - q_{\text{special}}^{\frac{l_1}{i_1}\cdot\frac{i}{l}}] \tag{2}$$

for beams with the ratio $l/i < \pi\sqrt{\frac{E}{S_{P\text{ or }E}}}$;

[3] *Cf.: Le Génie Civil*, Paris, August 24, 1935 and July 11, 1936; *ÖIA V*, Vienna, February 5, 1937; *La Génie Civil*, February 1, 1946 and November 15, 1948.

and the formula

$$F_{-1} = S_P[1 - q_{\text{special}}^{\left(\frac{l_1}{i_1}:\frac{i}{l}\right)^2}] \tag{3}$$

for beams with the ratio $l/i > \pi\sqrt{\frac{E}{S_{P\text{or}E}}}$.

The beams are fixed at both ends and may be subjected to millions of *reversals of compression and tension* or to *longtime steady compression longitudinally.*

q_{special} denotes the resilience q when the ratio $\frac{l}{i} = \frac{l_1}{i_1}$ of the beam equals $\pi\sqrt{\frac{E}{S_{P\text{or}E}}}$; l is the whole length of the beam which can vibrate longitudinally; i denotes the radius of gyration of the cross-section area; and E is the modulus of elasticity of the material.

For metals, whose stress-strain line curves gradually from S_P to the ultimate tensile stress or compression stress S_m, the resilience q_{special} equals $\frac{S_P}{S_m}$ and the corresponding F_{-1} is $\frac{S_m}{4}$ very approximately.

When $S_P \eqq 0.27S_m$, the limit of elasticity S_E of such metals can be computed by the aid of the equation

$$\frac{S_m}{4} = S_E\left(1 - \frac{S_P}{S_m}\right).$$

The resilience q_{special} of beams of steel can be computed very approximately by the aid of the equation

$$\frac{5}{12}S_m = S_P(1 - q_{\text{special}}).$$

Also for wooden beams the resilience q_{special} equals $\frac{S_P}{S_m}$, but for woods the values S_P and S_m for compression are mostly considerably lower than for tension. Some woods have lower or higher moduli of elasticity E for compression than for tension.

The new formulae (1), (2), and (3) give reliable endurance limits and do not need safety factors.

In consideration of the fact that the required cross-section area A of vertical beams fastened at both ends is computed by the aid of the equation

$$A = \frac{\text{steady load plus dynamic compression}}{F_{-1}},$$

it is suggested not to expect the beam with the chosen ratio $l/i_{min.}$ to take too much load plus dynamic compression, because in this case the required cross-section area A would be considerably bigger than the chosen cross-section area with the chosen $i_{min.}$ and often it would not be possible to increase the chosen cross-section area without increasing also $i_{min.}$

When only one end of the beam is fixed, but the other end fastened by a real joint, we have to use $\frac{1}{2} F_{-1}$ instead of F_{-1}, and when both ends of the beam are fastened by real joints, we must use $\frac{1}{4} F_{-1}$ instead of F_{-1} in the equation for A. For poles with their upper end free we must use $\frac{1}{16} F_{-1}$ in the equation for A. These reductions, which agree with the reductions of the critical load $\frac{4\pi^2 EJ}{l^2}$ for beams with both ends fixed, according to Euler, are necessary to avoid lateral vibrations and permanent bending of the beam being caused by one or two real joints or by the free end.

When the beam is a tube, then i^2 of the cross-section area equals $0.25(R^2 + r^2)$ and the cross-section area $A = \pi(R^2 - r^2)$ can be computed by approximate choice of R and r. The area A of a square cross-section is $12i^2$. The admissible load, to which the beam may be subjected, should not be higher than $F_{-1} \cdot A$ minus the dead weight of the beam.

In his treatise the author computed the admissible forces or load according to formulas 1, 2, and 3 for beams of metal and wood; furthermore, the critical loads $\frac{4\pi^2 EJ}{l^2}$ for beams with the ratio $l/i \leqq \pi \sqrt{\frac{E}{S_{P \text{ or } E}}}$.

Assuming the so-called safety factors to equal the ratio

$$\frac{\text{critical load } \frac{4\pi^2 EJ}{l^2} \text{ minus the deadweight of the beam}}{\text{admissible load } F_{-1} \cdot A \text{ minus the deadweight of the beam}}$$

the author computed these safety factors for very slender beams $\frac{l_1}{i_1} \cdot \frac{i}{l} = \frac{1}{9}$.

Given: S_P, S_m, E, l, i,

	Safety factor	
Plain Carbon Steel	5.4	tubelike beams.
Nickel Steel	7.3	
Nickel	5.1	
Duraluminum	4.5	
Oak	8.0	beams with square cross section area.
Fir	10.0	

The deduction of the deadweight of the beam from the critical load as given by Euler's formula or from the admissible compression $= S_P[1 - q_{\text{special}}^{\left(\frac{l_1}{i_1}\cdot\frac{i_{\min}}{l}\right)^2}]\cdot A$ (as given by the author) sets the uppermost limit for the utilization of vertical beams.

S_P, S_m, and E are determined in the known manner on test specimens of usual dimensions. For beams of metal with big thickness dimensions it is suggested to produce a specimen which has approximately the cross-section area of the actual beam and to test a strip cut from this specimen. However, sufficient safety is given also when S_P, S_m, and E are determined only on test specimens of usual dimensions, provided I E T L O beams with thickness dimensions not surpassing 0.8 in. are designed. Models and finished beams for actual service must be examined by X-rays or otherwise to ascertain their faultlessness.

Beams of metal and wood subjected to longtime steady tension longitudinally are subjected also to temporary vibrations. The endurance limit should therefore equal the original endurance limit which depends on the ratio l/i of the beam.

To sum up: Euler's and Tetmajer's formulae give reliable static endurance limits when the safety factor, or the "pure compression," is well estimated.

The author's formulae do not depend on *vague* forces or coefficients. They give reliable dynamic endurance limits which may be used also as static endurance limits. Last, but not least, the author's formulae do not need longtime tests.

SUPPLEMENT[4]

With regard to formulas 1, 2, and 3 above, the reader's attention is drawn to the fact that the exponential comparison ratios $\frac{l_1}{i_1} = \pi\sqrt{\frac{E}{S_P}}$ are to be used for beams subjected to longtime steady compression longitudinally and the ratios $\frac{l_1}{i_1} = \pi\sqrt{\frac{E}{S_E}}$ for beams subjected to millions of reversals of compression and tension.

Let us agree with Euler's formulae and assumption that the so-called critical load on a beam with fixed ends may be four times as high as the critical load on the same beam which has freely guided ends. Hence the endurance limit $F = S_P\,(1 - q)$ of the beam with fixed ends must be four times as high as the endurance limit of the beam with freely guided ends.

The possibilities for the intensity of the resilience q can be perceived from the author's fundamental formula

$$F = S_P(1 - q).$$

[4] This "Supplement," dated November 9, 1954, points out the advantages of the methods outlined above.

This formula makes it believable that for beams with the medium ratio $\frac{l_1}{i_1} = \pi\sqrt{\frac{E}{S_E}}$, the *medium* resilience can be $\frac{S_P}{S_m}$, provided the beam has fixed ends and the stress-strain-line of its material curves gradually from S_P to S_m. For instance, when the medium resilience $\frac{S_P}{S_m} = \frac{1}{2}$, then the resilience of the *same* beam, but with freely guided ends, can be computed from the equation

$$F = \frac{S_P(1 - \frac{1}{2})}{4} = S_P\,(1 - \tfrac{7}{8})$$

and amounts to $\frac{7}{8}$.

If we assume that the resilience $\frac{S_P}{S_m} = \frac{1}{2}$ refers to the beam with freely guided ends, then the endurance limit F_1, for the same beam, but with fixed ends, would be

$$4S_P(1 - \tfrac{1}{2}) = 2S_P$$

which is impossible.

One of the possible values of q for beams with freely guided ends is $\frac{4}{5}$, so that $F = S_P(1 - \frac{4}{5})$ and $F_1 = 4F = \frac{4}{5}S_P = S_P(1 - \frac{1}{5})$. But in this case the resilience $\frac{1}{5}$ would not be a medium resilience.

The correctness of this theoretical result has been confirmed by many long time tests of beams of light metals in rotating beam and Haigh-machines and also by longtime service of beams in mechanical and civil constructions.

In the years 1940 to 1946, in New York City, the author derived for beams of steel with fixed ends and with the ratio $\frac{l_1}{i_1} = \pi\sqrt{\frac{E}{S_E}}$, subjected to reversals of compression and tension, the formulas[5]

$$\text{Endurance limit } F_{-1} = \pm\frac{1}{2}\left[S_P + \frac{S_m - S_y}{2S_m - S_y}\,S_m\right].$$

The equation

$$F_{-1} = \pm\frac{1}{2}\left[S_P + \frac{S_m - S_y}{2S_m - S_y}\,S_m\right] = S_P\,(1 - q)$$

must be used for beams of steel in order to obtain the resilience q_{special} of a beam with fixed ends and with the ratio $\frac{l_1}{i_1} = \pi\sqrt{\frac{E}{S_{P\text{or}E}}}$. The value

[5] The value $\frac{5}{12}\,S_m$, cited on p. 337 of this paper, is an approximate value. For further details, see *Le Génie Civil*, Paris, February 1, 1946.

$\frac{S_P}{S_m}$ may not be used for beams of steel or other materials whose stress-strain line has a sharp break at the yield point.

The so-called critical load subjects the beam to compression and tension similarly as if it were bent by lateral forces.

When the actual load is much lower than the critical load, then the very straight and faultless beam can take longitudinal uniform compression without being impaired by lateral vibrations.

However, when the actual load is not sufficiently lower than the critical load, then *any eccentric longitudinal force and the unavoidable lateral vibrations can cause creep* in straight and faultless beams even when the ratio $\frac{l_1}{i_1}$ is less than $\pi \sqrt{\frac{E}{S_{P \text{ or } E}}}$. When the compression becomes unsymmetric and when it drops to zero in one part of the surface of the beam, then this part is subjected to reversals of compression and tension by lateral vibrations and the actual load becomes a critical load gradually.

However, when the beam is profiled with such a cross-section area that the admissible stress per unit cross-section area equals the endurance limit F_{-1} for reversals of compression and tension, then moderate lateral vibrations of the beam cannot impair the beam. Therefore the resilience q_{special} of a beam with fixed ends and with the ratio $\frac{l_1}{i_1} = \pi \times \sqrt{\frac{E}{S_P}}$, when this beam is subjected to a steady load and to *lateral vibrations*, equals the resilience q_{special} of the same beam, when this beam is subjected to reversals of longitudinal compression and tension.

When we use Euler's formula $P = \frac{\pi^2 EJ}{l^2}$, then we need a safety factor. This factor is chosen arbitrarily in different countries.

For ratios $\frac{l}{i_1} < \pi \sqrt{\frac{E}{S_E}}$ Professor L. von Tetmajer has given the empiric formula

$$\frac{\sigma_k}{\sigma_d} = \frac{\beta_k}{\beta_d} = 1 - a \frac{l}{i}$$

for steels and wood.

According to this formula the endurance line (geometric locus of the endurance limits) would run as a straight line downward close to $\frac{l}{i} = 0$. Hence, the equation $\beta_k = \beta_d \left(1 - a \frac{l}{i}\right)$ would mean that crush cannot be caused unless l nearly equals zero. Experiments show, however,

that though the endurance line runs downward as an extremely flat curve, it has a turning point at about $\frac{1}{4}\frac{l_1}{i_1}$. Beams with a very low ratio $\frac{l}{i}$, as a matter of fact, can be crushed. Professor L. Von Tetmajer, as a great experimenter, of course knew this fact and in his tables has given $\frac{\beta_k}{\beta_d}$ only for ratios from 15 upward for "Flusseisen" and steel and for ratios $\frac{l}{i}$ from 5 upward for wood (without mentioning the kind of wood).[6]

Tetmajer's equation

$$\sigma_k = \sigma_d \left(1 - a\frac{l}{i}\right)$$

requires longtime tests for the determination of σ_d, a vague vector which was called by Prof. Tetmajer "pure compression" and estimated to be $\frac{S_m}{4}$ for steels, longtime tests also for the determination of the coefficient a.

Tetmajer's formula was thought to be used both for beams with loosely guided ends and for beams with fixed ends which have the same length. This causes difficulties with regard to the reduction of the length to be taken into account.

The author's formula

$$F_{-1} = S_P[1 - q_{\text{special}}^{\frac{l_1}{i_1}\cdot\frac{i}{l}}]$$

for ratios $\frac{l}{i} < \pi\sqrt{\frac{E}{S_P}}$ or $\frac{l}{i} < \pi\sqrt{\frac{E}{S_E}}$ was obtained from many test results. The line, which illustrates the values of the resilience for variable ratios $\frac{l}{i}$, runs as an extremely flat curve at either side of the resilience q_{special}. No break nor turning point exists till *about* $\frac{l}{i} = \frac{1}{4}\pi \times \sqrt{\frac{E}{S_{E\text{ or }P}}}$.

The course of the resilience line for $\frac{l}{i} < \pi\sqrt{\frac{E}{S_{E\text{ or }P}}}$ is best represented

[6] *Beams* of steel with $\frac{l}{i} > 15$, and beams of wood with $\frac{l}{i} > 5$ can be bent even when $\frac{l}{i} < \pi\sqrt{\frac{E}{S_{P\text{ or }E}}}$.

by the variable expression $q_{\text{special}}^{\left(\frac{l_1}{i_1}\cdot\frac{i}{l}\right)}$ in the author's formula:

$$F_{-1} = S_P\left(1 - q_{\text{special}}^{\frac{l_1}{i_1}\cdot\frac{i}{l}}\right)$$

because the exponent $\frac{l_1}{i_1}\cdot\frac{i}{l}$ gives the same slope of the resilience line downward from q_{special} as the exponent $\left(\frac{l_1}{i_1}\cdot\frac{i}{l}\right)^2$ causes upward from q_{special}.

The admissible longitudinal compression as computed by the author's formula may be divided by four in order to obtain the admissible compression of the same beam with the same length but with real links at its ends instead of fixed ends.

For network constructions with riveted knots we suggest taking into account for compression-members the length l from innermost rivet to innermost rivet in the unchanged formulae 1 and 2 given above.

Network constructions with knots performed by the aid of gusset-plates and rivets are subjected to secondary stresses which increase when the lateral deflection of the whole network is increasing.

Bridges with trusses whose free span is longer than ten times the height (= width) of one field, are subjected to considerable secondary stresses.

For beams of plain carbon steel with ratios $\frac{l}{i} = 135$ to $\frac{l}{i} = 90$, the author's formulae give 20 to 40 per cent lower endurance limits than the combined Euler-Tetmajer formulae. The lower endurance limits require bigger cross-section areas of the beams and thus reduce the lateral deflection and consequently the secondary stresses of the whole network to a minimum.

Attention is drawn to the fact that for steels the limit of elasticity S_E mostly does not lie as high over S_P as it does for metals whose stress-strain line curves gradually from S_P to S_m.

When $S_P \lessapprox 0.27S_m$, the limit of elasticity S_E can be computed by the aid of the equation

$$\frac{S_m}{4} = S_E\left(1 - \frac{S_P}{S_m}\right)$$

and may be used instead of S_P as the first factor in formulae 1, 2, and 3.

When $S_P \gtrapprox 0.27S_m$, and the stress-strain line curves gradually from S_P to S_m, then the true yield proint (S_y) can be determined by a simple graphic method and S_E equals at least $\frac{1}{2}(S_P + S_y)$.

In housings, the vibration of columns by longitudinal steady compression plus minor dynamic impulses can be caused by persons dancing over the columns and by other influences. Posts in the open air are subjected to vibrations by wind pressure.

Steady compression plus dynamic impulses causes vibrations also in airplane wings, especially when the plane is flying downward.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS

ABRASIVE JET METHOD FOR EVALUATING ORGANIC COATINGS

A simple, rapid method for measuring the abrasion resistance of organic coatings on metals has been developed by A. G. Roberts, W. A. Crouse, and R. S. Pizer of the National Bureau of Standards. Essentially, the method determines, under controlled operating conditions, the time required for a high-speed jet of fine abrasive particles to abrade through the coating to the substrate. Development was sponsored by the Navy Bureau of Aeronautics in connection with a program for the production of highly durable, scuff-resistant finishes for naval aircraft. Because the instrument readily simulates a variety of service conditions, the NBS procedure can be used to determine abrasion resistance on all types of protective coatings, regardless of gloss, color, thickness, or surface area.

Abrasion resistance is often one of the most important factors in the service performance of protective coatings. Until recently thoroughly reliable methods for determination of this property have been lacking. Several abrasion test procedures are available in which loose particles fall, rub, or are blown against the test specimen. Although the NBS abrasive jet method is similar in principle to these methods, it permits greater ease and rapidity in evaluating materials and better reproducibility because it does not depend upon abrading an area of definite size and uses a continuously fresh supply of abrasive particles under closely controlled conditions. The end point in this method is the first show of bare metal and is readily detected when coating and substrate differ in color, as is usually the case; otherwise, inspection with a hand lens can be used.

In the abrasive jet method, carbon dioxide gas under controlled pressure propels an abrasive powder from a vibrating storage chamber through a nozzle to impinge at high speed upon the test specimen. A solenoid valve operated with a manual switch turns the flow of the abrasive powder on or off. The mounting arrangement for the test specimen and nozzle assembly permits rotational, back and forth, or up and down adjustment, while the angle between the nozzle tip and surface to be abraded is adjusted by a graduated rotating disk attached to the nozzle shaft. A specially calibrated taper gage permits rapid and accurate adjustment of the nozzle-to-specimen distance at abrading angles from 20° to 90°. The specimen holder and the nozzle assembly with its adjusting parts are housed in a rectangular brass chamber with a transparent plastic front door and top.

The spent abrasive particles and coating debris are removed through a vacuum exhaust outlet at one end of the test chamber. Rapid monitoring of the abrasive flow rate is accomplished by a weighing tube that has a rubber diaphragm with a hole to accommodate the nozzle tip at one end and a diaphragm of ordinary filter paper at the other end to permit escape of gas while trapping the abrasive particles.

Although operating conditions can be varied to accommodate extreme differences in materials, it is usually desirable to use a nozzle-coating distance of 0.04 in., a gas pressure of 40 psi., a flow rate of approximately 0.15 g. per second, and abrading angles of 90° or 45°. The 90° angle simulates the leading-edge type of erosion encountered by aircraft in flight, while the 45° angle simulates scuffing wear which aircraft receive during maintenance operations when they are walked on, struck, or scraped by hose nozzles.

The abrasion time for a given material depends on the test conditions; it increases almost linearly with increasing nozzle-to-coating distance, decreases with increasing pressure, and decreases with decreasing angle of abrasion from 90° down to about 30°. At smaller abrading angles, the abrasion time rises again.

In the studies at the National Bureau of Standards, the abrasive jet method has been found to distinguish significantly between materials difficult to differentiate by qualitative means. Although the degree of correlation with actual service performance has not yet been definitely established, work toward this end is now in progress.

FROST CLOSURE OF ROOF VENTS IN PLUMBING SYSTEMS

In the northern part of this country and in Canada, plumbing system roof vents sometimes become partially or completely closed by frost accumulations during very cold weather. Various practical expedients for preventing this phenomenon have been tried, some of which have been successful. For example, some plumbing codes specify a 4 in. minimum diameter for the exposed portion of a roof vent. However, no scientific method for predicting the extent of frost closure under given conditions appears to have been developed until H. N. Eaton and R. S. Wyly of the National Bureau of Standards conducted an investigation of this problem. The study was sponsored in part by the Housing and Home Finance Agency.

The principal objection to frost closure is the effect created by excessive reduction of the cross section of the roof vent. When this happens, air flow into the vent cannot completely replace the air that is carried down the drainage stack and out of the plumbing system when water is discharged from the fixtures. If the roof vent is nearly or completely closed, the discharge of a single water closet will often suffice to siphon the water seals from one or more fixture traps on the system,

thereby making it possible for sewer atmosphere to enter the rooms of the building in which the plumbing system is located.

Frost closure occurs naturally in two forms: (1) A cylindrical layer of frost on the inside of the exposed portion of the vent pipe extending above the roof, and (2) a frost cap at the top of the vent. The two forms may occur simultaneously.

The moisture that produces the frost layer or frost cap comes from the stream of warm moist air that ordinarily flows upward through the drainage stack and roof vent in cold weather as a result of chimney action. The column of warm air inside the stack and vent weighs less than a similar column of cold air outdoors; thus the air inside the system rises except when plumbing fixtures are being discharged, at which time the direction of air flow is temporarily reversed.

The exposed part of the roof vent is chilled by the outside air. Hence, as the current of warm moist air rises through the vent, this air is cooled enough so that part of its moisture condenses. Some of the droplets of condensed moisture are carried out of the vent with the air stream while others strike the wall, where they freeze if the temperature of the inner surface of the vent is below the freezing point of water. Thus the water that freezes on the wall of the vent builds up a layer of frost or ice that increases in thickness as long as the temperature of its inner surface remains below 32 F.

Because the temperature of the air stream in the roof vent is higher than that of the outside air, heat flows radially outward from the vent. According to the NBS study, this transfer takes place in four distinct steps: (1) From the air stream in the vent to the ice-air surface; (2) from the ice-air surface through the ice layer to the inner wall of the vent; (3) through the wall of the vent; (4) from the outer wall of the vent to the atmosphere. An analysis of this process of heat transfer indicates that the heat liberated by the freezing of the water droplets must be taken into account. The velocity of the air stream in the vent and of the wind blowing across the roof must also be considered as well as the thickness and thermal conductivity of the ice layer and the vent. It has been commonly believed that if the weather is cold enough to initiate the process of frost closure in a vent of any diameter and that if these conditions persist for a sufficiently long period, the vent will become completely closed, but this is not always true. Actually, under some conditions a layer of ice or frost forms on the wall of the vent and increases in thickness up to a certain maximum value, depending on the diameter of the vent and on the rate at which heat is extracted from the air stream through the wall of the vent. This thickness may or may not be sufficient to close the vent.

By applying the heat transfer analysis to a set of assumed conditions, it is possible to compute the temperature of the ice-air surface as the vent freezes. For a 4-in. metal vent, a wind velocity of 10 ft/sec across

the roof, an air stream temperature in the vent of 60 F., and an outside air temperature of less than about 7 F., the vent will freeze solid, but for outside temperatures greater than about 7 F. it will not. For example, if the outside temperature is 10 F., the computations indicate that freezing will stop when the diameter of the opening is reduced to about 0.63 of its original value, while for an outside air temperature of 20 F. no reduction whatever will take place. Although the analysis indicates the controlling conditions and the extent of frost closure, it does not show the length of time required to close the vent opening to any particular size.

Frost closure has been produced experimentally in the Bureau laboratories. Roof vents of several different diameters were extended up into a chamber in which temperatures as low as − 30 F. were maintained, and warm moist air was blown up through the drainage stack and roof vent to simulate the conditions that cause frost closure in actual service. Periods in excess of 9 hours were generally required to close 1½-in. vents exposed to a temperature of − 30 F., and increasing the diameter of the vent or decreasing the length of vent exposed to the low temperature increased the time required for effecting complete closure.

Recorded observations of frost closure under service conditions in Canada for 6- and 8-in. vents were found to agree rather closely with the predictions of the Bureau's analysis. The Canadian field data are limited in scope and some of the conditions which may have determined the results are not known; therefore these data probably do not provide a sound basis for judging the degree of accuracy of predictions made by theoretical means. However these data indicate that the method used in the analysis should give an approximation of the extent of frost closure in service under various conditions. The study also indicates the desirability of keeping the extension of the vent above the roof as short as practicable, of increasing the diameter of the roof vent one or two sizes over the diameter of the stack, and of using insulating materials around the roof vent in areas where extremely cold weather may be expected.

NOTE: For further technical information, see "Frost Closure of Roof Vents in Plumbing Systems," NBS Building Materials and Structures Report 142, available from the Superintendent of Documents, Government Printing Office, Washington, D. C. for 25 cents.

NUCLEAR ENERGY NEWS

REACTOR FOR BATTELLE INSTITUTE

A contract for the design, engineering, and construction of a nuclear reactor has been awarded by Battelle Institute, Columbus, Ohio, to the American Machine and Foundry Company, New York.

The reactor will be a key component in a $1\frac{1}{2}$-million-dollar atomic research center Battelle is building fifteen miles west of downtown Columbus. This reactor will be one of the first in the country to be owned and operated by a private organization.

The reactor will be designed to be of broad use in the study of problems of industry by providing an intense source of neutrons and atomic radiation for research uses. It will supplement a large cobalt-60 gamma radiation source now being installed by Battelle, and reactor development and hot-cell laboratories planned by the Institute.

One of the most important uses of the reactor will be to furnish neutrons for conducting exponential experiments on power-reactor cores. Such experiments are necessary in the development of nuclear plants for the generation of electricity.

The reactor will also be used in Battelle's research for the chemical, petroleum, metals, machinery, ceramics, electronics, food processing, textile, and pharmaceutical industries. Its intense radiation will make possible new explorations in the field of radiation-activated chemical reactions. It will provide radioisotopes for tracer studies in agriculture, medicine, wear and lubrication, and many other fields of science and technology. An important application will be in the study of the effects of radiation on engineering materials that will be used in atomic engines.

The reactor, together with auxiliary equipment and the building in which it is to be housed, will cost an estimated one-half million dollars. Design work will start immediately. Construction will begin after review and licensing by the Atomic Energy Commission. The target date for "going critical" (starting the nuclear fission reaction) is February 1, 1956.

Actively engaged in atomic energy development since the earliest days of the United States atomic energy program, the American Machine and Foundry Company has designed and developed a variety of special atomic energy equipment. The company has been especially engaged in reactor design and in the development and manufacture of reactor-control and remote-handling systems. American Machine and Foundry was one of the first companies to join on an individual basis

the Atomic Energy Commission's industrial participation program for the development of competitive nuclear power.

The reactor will be designed to operate at 1000 kilowatts on uranium-235 fuel. It will be of the so-called "swimming-pool" type, which means that much of the shielding will be provided by immersing the core in a pool of water, where it is much more accessible for experimentation than if encased in lead and concrete. Its core will be similar to that in the Materials Testing Reactor at the National Reactor Testing Station in Idaho.

VAN DE GRAAF GENERATOR AT COLUMBIA UNIVERSITY

The U. S. Atomic Energy Commission will install a Van de Graaff generator at Columbia University capable of accelerating particles to energies of six million electron volts. The accelerator is being made by the High Voltage Engineering Corp., Cambridge, Massachusetts. Cost of the machine plus auxiliary equipment and installation will total about $450,000. The AEC will also contribute approximately $295,000 toward a laboratory to house the machine.

The installation of this accelerator will mark a step forward in the Commission's program of nuclear cross-section measurements. The staff at Columbia University has been deeply involved in the measurement of basic nuclear properties and cross-sections since well before World War II. Through these efforts, which have been carried out under the direction of Professor William W. Havens, substantial contributions have been made toward obtaining the basic nuclear data needed for atomic reactor and weapon design.

PATENTS RELEASED FOR PUBLIC USE BY AEC

The Atomic Energy Commission has released a group of 21 patents granted during the fourth quarter of 1954 and owned by the U. S. Government. They deal principally with apparatus and equipment used in the Atomic Energy program.

The AEC will grant non-exclusive, royalty-free licenses on these patents, as part of its program to make non-secret technological information available for use by industry. Commission-held patents and patent applications released for licensing now total 747.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

March 16, 1955

The Stated Monthly Meeting of The Franklin Institute was held on March 16, 1955, in the Lecture Hall. S. Wyman Rolph, President, called the meeting to order at 8:15 P.M. In the absence of the Secretary, H. V. Bail, Assistant Secretary, kept the Minutes of the meeting. Approximately 415 members and guests were in attendance.

The President stated that the Minutes of the Annual Meeting in January were printed in the February JOURNAL and asked for additions, corrections, or changes. There being none, they were, on motion duly made and seconded, approved as submitted. He advised that Minutes of the February Stated Meeting will be printed in the March JOURNAL and will be submitted for approval at the next Stated Meeting.

He reported that elections to membership, recorded since January 31st, resulted in a net gain of 3 Student Members, with a total membership as of February 28th of 6,817.

The President then welcomed the officers and members of the Philadelphia Society for Promoting Agriculture to their fifteenth joint meeting with The Franklin Institute and introduced J. Gordon Fetterman, President of the Society, who took the chair. Following a warm welcome extended to the speaker by Walter M. Phillips, City Representative, Mr. Fetterman introduced the speaker for the evening—The Honorable Ezra Taft Benson, Secretary of Agriculture. The Secretary's talk on "American Agriculture—A Great Past, A Greater Future" received a spontaneous and prolonged ovation from the audience. He was then awarded the Society's annual medal for outstanding work in the science of agriculture. Mr. Benson's address will be published in the July issue of the JOURNAL.

After a question period, the meeting adjourned at 9:40 P.M. with a rising vote of thanks to Mr. Benson.

H. V. BAIL
Assistant Secretary

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

SWANN, W. F. G.: The Known and the Unknown

FAUPEL, J. H.: Residual Stresses in Heavy-Wall Cylinders

FALES, ELISHA N.: A New Laboratory Technique for Investigation of the Origin of Fluid Turbulence

RYDER, FREDERICK: Linear Algebraic Computation by Multi-Winding Transformers

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, March 9, 1955.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 9, 1955.

MR. RALPH S. MCCLARREN in the Chair.

The following reports were presented for final action:

No. 3261: Séjournet Extrusion Process for Metals.

This report recommended the award of a John Price Wetherill Medal to Jacques Yves Pierre Séjournet, of Persan, France, "In consideration of his invention of the Ugine-Séjournet Extrusion Process for metals—stressing the ingenious utilization of glass as a lubricant and the effective method of its use."

No. 3285: Brown Medal.

This report recommended the award of the Frank P. Brown Medal to Charles S. Leopold, of Philadelphia, Pennsylvania, "In consideration of his outstanding contribution to air conditioning, particularly in the research and development of air conditioning techniques and the application of air conditioning to a wide variety of important structures."

JOHN FRAZER,
Secretary to Committee

MEMBERSHIP

ACTIVE MEMBERS ELECTED BY THE BOARD OF MANAGERS

March 16, 1955

ACTIVE FAMILY

James W. Bampton — James R. Mays — Raymon C. Northamer

ACTIVE

Robert C. Bucher
Anton Buchert
Frank R. Eltinge
George English
James J. Flynn
Martha A. Gable
C. Edgar Hires
John LaCerda
James P. Magill
J. Walter McCall
Howard H. McChesney
R. W. McIntosh
Morton S. Mitchell
Charles F. Palmer, Jr.
Arthur C. Richards
H. Radclyffe Roberts
S. Eugene Sall
Donald M. Simons
F. M. Suplee
H. B. Walker
Louis H. Watmough

ACTIVE NON-RESIDENT

Peter Paul Kellogg — L. W. McCormick

NECROLOGY

James O. Anderson, '46
Harry C. Carr, '45
J. R. Downes, '45
L. P. Fessler, '51
G. Krauss, '53
Alan H. Miller, '50
John H. Neilson, '51
Mrs. Frank A. Wills, '43
Hamilton M. Wilson, '34

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A. M. until 5 P. M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; 2 P. M. until 10 P. M. on Wednesdays.

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TO MEMBERS AND THOSE WHO KNOW THE FRANKLIN INSTITUTE

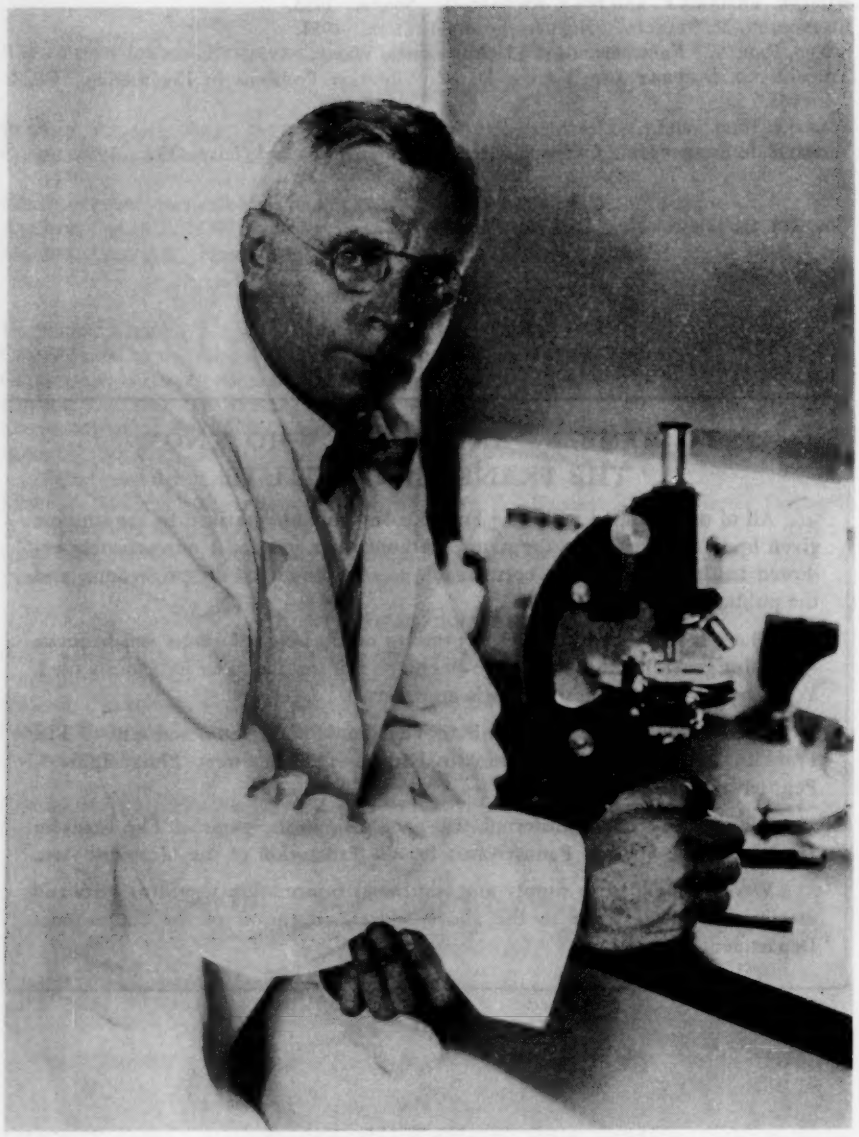
All of us associated with The Franklin Institute are gratified by the support given by its benefactors in the past. Without their generosity our privately endowed Institute would have been unable to serve industry, the professions, and the public as it has and should.

There is still a continuing and pressing need of contributions and bequests —both for general and specific endowment, and of income to be judiciously used. Your response will be most earnestly appreciated.

Checks should be made payable to The Franklin Institute, and sent to The Franklin Institute, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pennsylvania.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

We shall be glad to supply any additional information regarding gifts and memorials. Please write to the above address, attention of the Endowment Department.



DR. ELLICE MCDONALD

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, FOUNDER

The death of Dr. Ellice McDonald at his home in Delaware on January 30, 1955 meant the loss of a pioneer in the field of cancer research. He was the Director of the Biochemical Research Foundation of The Franklin Institute since its inception in 1935.

To many who knew Dr. McDonald in recent years, the names Ellice McDonald and Biochemical Research Foundation were synonymous. He himself often said that the modern research institution was the elongated shadow of one man. Not only was the Foundation his conception, it was his life for the past several decades.

That was not to say, however, that it was his sole interest. His interests in medical research and in the medical world were extensive. After receiving his M.D. degree from McGill University, Dr. McDonald served as resident surgeon at the Kensington Hospital, Philadelphia, and New York Lying-in Hospital; then as assistant in pathology, Albany Medical School; and as instructor at Columbia University College of Pharmacy and Science.

He was instructor in surgery at the New York Postgraduate School and Hospital from 1907 to 1916. During that time Dr. McDonald introduced a solution for hand sterilization which became known as McDonald's solution, and published a volume of "Studies in Gynecology and Obstetrics."

From 1922 to 1935 he was assistant professor of gynecology at the Graduate School of Medicine of the University of Pennsylvania. It was during this period that he translated his interest and ideas concerning cancer, the investigation of its cause and possible cure, into a definite plan for cancer research. In 1927 he organized the Cancer Research Laboratory of the Graduate School of Medicine at the University of Pennsylvania in Philadelphia, and was its Director.

Never afraid to try any new tool, method, or idea for use in the attack upon the cancer problem, he was one of the first to co-ordinate the efforts of the chemist, the physicist, the biologist, the pathologist, in the study of cancer. His pattern for the correlation of effort by scientists of the different disciplines has served as a model for the now established organizations devoted to cancer research.

During the eight years of the existence of the Cancer Research Laboratory it became evident to Dr. McDonald that in order to learn more about cancer it was necessary to learn more about non-cancerous and precancerous biochemical processes. In 1935, therefore, by reorganization and re-orientation, the Cancer Research Laboratory be-

came the Biochemical Research Foundation of The Franklin Institute, with Dr. McDonald as Director. The major emphasis of the work was still on cancer research, perhaps to an even greater degree.

When it became evident that additional working space was required, it was decided to move the Biochemical Research Foundation to Newark, Delaware. Dr. McDonald himself helped in every detail in the planning for the building of the new laboratory. After the removal of the entire organization, in 1940, he planned and built a home in Delaware in order to be close to the laboratory.

The new building made it possible to have under one roof all of the research equipment and apparatus, including the cyclotron, which was the first one to be built for the study of the effect of artificially generated high energy particles upon biological systems. During the existence of the Manhattan Project, the cyclotron at the Foundation was used primarily to determine the radiological effects of neutrons upon living systems. This work was published in 1947 in a volume titled "Neutron Effects on Animals."

Dr. McDonald's faculty of a well developed memory helped him in the integration of current research both at home and abroad. He was responsible for bringing well known specialists and their assistants to this country, introducing their methods, and blazing the pathway for new approaches.

His organization and development of cancer and biochemical research sharpened Dr. McDonald's interest in the management of research. His brochure on "Research and its Organization" was widely read and requests for copies were received from countries throughout the world.

Dr. McDonald was born October 27, 1876 at Fort Ellice, Canada. His father was the last Chief Factor of the Hudson's Bay Company and Dr. McDonald took keen interest in the history of that company. Because he devoted so much of his time, effort and thought to his beloved research, he must have seemed to those who met him to have one major interest and one only. But those who came to know him better were constantly amazed at his many interests and the many, not directly related subjects of which he had more than casual, passing knowledge. It can be truly said of his many interests that he loved not them the less, but Cancer Research the more.

* * *

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, FOUNDER

ANAEROBIC GLUCOLYSIS OF THE KREBS ASCITES CARCINOMA ON THE BASIS OF CELL NUMBER

BY

W. D. YUSHOK

The investigation of metabolism of cancer cells has been greatly facilitated by the development of the ascites form of certain animal tumors (1, 2, 3). The Krebs 2 carcinoma (4), as well as several other malignant tumors, grows rapidly in the peritoneal cavity of the mouse as a suspension of individual cells. It forms a nearly pure culture of cancer cells which can be readily separated from the ascitic fluid by centrifugation and resuspended in any buffer or nutrient medium desired. It is amenable to precise quantitative measurements of metabolic processes and their inhibitions to a degree that is not possible with slices of solid tumor tissue. Furthermore, its metabolic activities can be determined on the basis of a known cell population.

The rate at which glucose is converted to lactic acid by the Krebs ascites tumor under anaerobic conditions has been determined on the basis of a population of one million cells. Also, the rate of glucose utilization has been calculated on the basis of the cell, the smallest biological unit capable of sustaining life.

MATERIAL AND METHODS

The Krebs 2 ascites carcinoma was obtained from Dr. T. S. Hauschka of the Institute of Cancer Research. The tumor strain was propagated in female white Swiss mice by intraperitoneal injection of 0.1 ml. of the ascites tumor material. For the metabolic tests, ascitic fluid was withdrawn from 2 to 4 animals 4 to 7 days following inoculation.

The ascitic fluid was diluted with cold Krebs-Henseleit bicarbonate (KH) buffer previously gassed with a mixture of 95% N_2 and 5% CO_2. The mixture was spun in a refrigerated centrifuge at 1000 *g* for a 1-minute period. During this short period of centrifugation at low speed the red blood cells, if present, remained suspended in the supernatant fluid; most of the tumor cells were spun down, probably because of larger size or symmetrical shape. The tumor cells were resuspended in buffer and centrifuged again under the above-mentioned conditions for a 1-minute period to remove any remaining red blood cells and extracellular materials. The cells were again resuspended in buffer and were centrifuged for a 3-minute period to assure gentle but firm packing of the cells.

Approximately 1 ml. of pooled packed cells was suspended in 50 ml. KH buffer. Into the main chamber of each Warburg vessel, 2 ml. of this cell suspension were pipetted. D-Glucose was dissolved in KH buffer and was pipetted into the main chamber. KH buffer was then added to bring the total fluid volume up to 3 ml. The manometric system was gassed for 8 minutes with a mixture of 5% CO_2 and 95% N_2 (prepurified). The system was equilibrated for 7 minutes before the manometric readings were begun.

For cell counts, 2 ml. of the cell suspension were diluted to 25 ml. in each of 2 volumetric flasks with physiological saline solution. Counts were made in duplicate by means of a hemocytometer.

Independent determinations of dry and wet weights were made of known numbers of cells. Each cell suspension was pipetted into a tared microcentrifuge tube. The cells were centrifuged at 2500 *g* for 30 minutes, were weighed, dried overnight at 105° C., and were weighed again.

RESULTS AND DISCUSSION

The mean value of the maximal glucolytic rate in the presence of 15×10^{-3} M glucose was 22.5 μl. CO_2 per million cells per hour at 37.5° C. The range of experimental values of 21 different cell suspensions prepared on different days was 18.4 to 25.8. In the absence of glucose, the rate was 0.17 μl. CO_2 per million cells per hour. This control value is only 0.8 per cent of the maximal rate of glucolysis.

Since one-half mole of glucose produces one mole of lactic acid which in turn releases one mole of CO_2 to the atmosphere in the manometric system, the maximal glucolytic rate is equivalent to the utilization of 0.501 micromoles of glucose per million cells per hour. Therefore, one Krebs cell which is representative of the cell population has the capacity under anaerobic conditions to convert 84 million molecules of glucose per second to lactic acid.

The ability of the cancer cells to metabolize glucose at a rapid rate may be shown in relation to their weight. It was calculated that Krebs cells have the capacity to metabolize their weight of glucose in approximately 31 hours. These cells can split a quantity of glucose equal to their dry weight in about 5 hours.

The range of glucolytic rates of the Krebs cells was calculated on a milligram dry weight basis to be 40 to 56 μl. CO_2 per hour ($Q^{N_2}_{CO_2}$). Warburg found that the $Q^{N_2}_{CO_2}$ of the Ehrlich ascites carcinoma was 45 to 55 (5). The glucolytic capacities of these two ascites tumors are significantly higher than those of tumor tissue slices in which the $Q^{N_2}_{CO_2}$ is 13 to 37 (6). This lower rate is attributed to the presence of connective tissue, blood capillaries, and dying or dead cells in the solid tumor. The presence of injured and cut cells in the tissue slice is unavoidable. The tumor slice usually produces a significant quantity of acid even in

the absence of added substrate. In contrast, the ascites cells, which can easily be washed free of extracellular material, do not produce a measurable amount of acid unless a glycolyzable substrate is added. The ascites tumor, therefore, has been found to be an excellent material for studying the rates of anaerobic metabolism and for screening chemicals as possible inhibitors of this system in cancer cells.

SUMMARY

The anaerobic glucolysis of the Krebs ascites carcinoma has been determined by the Warburg manometric method and expressed on the basis of cell number.

It has been calculated that one average Krebs cell has the capacity under anaerobic conditions to metabolize to lactic acid 84 million molecules of glucose per second.

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NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

A RADIATION PYROMETER

BY

JOHN H. KRATZ

Several months ago, the staff of the Chemical Kinetics and Spectroscopy Section, Chemistry and Physics Division undertook a project for the Philadelphia Electric Company. The problem: to measure the temperature of the short-circuiting-bars, of the amortisseur winding, on a 30,000 KW single-phase, 25 cycle generator of a frequency convertor while running under full load conditions.

There are ten of these bars, or short-circuiting segments, in the amortisseur winding; each is 44 in. long and they form a ring 37 ft. in circumference. Since the ring rotates with the generator at 300 rpm., and because there is a large temperature gradient along the length of each bar, the measuring device had to be able to follow changes in temperature which occur in less than 1/1000 of a second.

The bars are 3 ft. from the outside housing of the machine and are visible only through a narrow opening between the fixed stator windings and the pole pieces. The motion of the bars, magnetic field, high voltage, and heat further limited the choice of instrumentation.

To meet these requirements, a radiation pyrometer was designed around a lead sulfide photoconductive cell (Ektron cell) as the detector. The lead sulfide cell has an adequate frequency response and, with a suitable optical system, could be placed outside of the generator on a concrete floor 7 ft. from the bars.

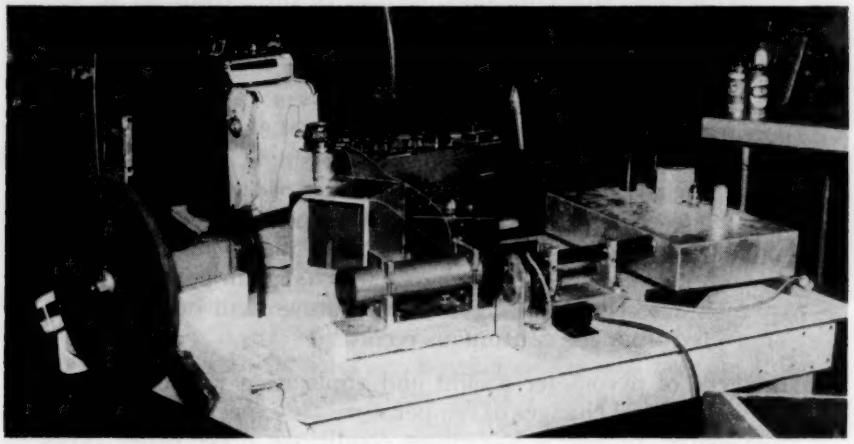
The optical system is a collimator composed of a quartz lens and stops. It receives only the radiation from a 2-in. spot on the amortisseur bar and focuses it on the lead sulfide detector.

With this equipment, it is possible to make continuous measurements of temperature as well as measurements of average temperature.

CONTINUOUS MEASUREMENT OF TEMPERATURE

One method of operation results in a continuous oscillograph record of temperature along the length of the bars. An 8-ft. length of film, which is run off in approximately two seconds, shows the temperature of the bars during about ten revolutions of the machine. In this mode of operation the Ektron cell feeds an a-c. amplifier which amplifies the signal to a level suitable for operation of the oscillograph.

To calibrate the instrument during a recording run, a time-sharing system is used consisting of a disc with three mirrors on its circumference, which rotates at 240 rpm. in front of the optical system. Through the spaces between mirrors, the optical system "sees" the amortisseur bars, but when the mirrors interrupt this view the optical system sees a standard source of radiation. Each mirror takes 1/50 of a second to pass completely through the field of view of the pyrometer, which is the same length of time each bar takes. Timing is such that in two revolutions of the disc all the bars and all the calibration pulses show up on the



The radiation pyrometer and associated equipment. Left to right: disc with three mirrors, recorder, standard source, optical system (foreground), and chassis containing the necessary circuits.

record. Two of the three mirrors have screen material over them, which lessens the amount of energy reflected to the detector. The two screens are of different mesh, so that in combination with the standard source of radiation (the third, unscreened, mirror) they represent two temperatures to the detector. One rotation of the mirrored chopper injects three temperature calibrating pulses into the record.

MEASUREMENT OF AVERAGE TEMPERATURE

This type of operation uses a chopper or synchronous shutter which allows the Ektron cell to see only a 2-in. circular spot on each of the ten bars. This position can be changed by a phase adjustment of the synchronous motor which drives the chopper; the ten spots can be at the leading edge, or at any location along the bars, or even at the space between the bars. This signal is amplified, rectified, and fed into a capacitor which integrates the signal, giving a d-c. output that varies with temperature. This d-c. voltage represents the average temperature of the ten spots and is fed into an amplifier which drives an Esterline-Angus recorder.

The Esterline-Angus record is calibrated by rotating the mirrored chopper by hand so that each of the three mirrors is in the beam long enough to record its representative temperature. This was done before and after each average-temperature run.

Records of temperature were made at different loads and power factors on two machines, along with records of current, voltage, air temperature, etc. The measurements were made after the converter had operated at high load levels for 12 hr. so that heat equilibrium would be reached inside the machine. This produced room temperatures of 140 F. and necessitated controlled cooling of the pyrometer to 100 F. for greater sensitivity and stability. The data obtained with the pyrometer and evaluated by Philadelphia Electric Company engineers showed:

1. temperatures in the range of 200 to 300 F. (differentials of as much as 80 F. were measured across a single bar).
2. comparison of the temperatures of bars of two different designs,
3. the time constant for a change in temperature resulting from a change in load,
4. a shift of a hot spot on the bars with change in power factor, and
5. that a defective bar (higher temperature than others) could be detected from the continuous record.

This type of pyrometer should find application where parts are in motion, where rapid changes of temperature are expected, where objects are inaccessible, or where hazardous conditions make other types of pyrometers unsuitable.

Acknowledgement

Acknowledgment is hereby made of the assistance in the development and application of the radiation pyrometer given to Laboratories personnel by the Philadelphia Electric Company.

NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

THE INTERNAL COMBUSTION ENGINE

It may seem incredible that every man, woman, and child in the United States has 39 "horses" working for them. That is only another way of saying we have over six billion horse-power harnassed for our use. In the last twelve months we used more power than was used during the first seventy-five years after the nation was founded.

The interesting feature about this vast accumulation of power is that by far the greater proportion is provided by internal combustion engines, and not all of these are under the hoods of automobiles.

The history of the internal combustion engine is of great interest for the theory of invention, because basic principles were perceived at early dates, although substantial practical application was relatively late. There were critical problems of design that obstructed final achievement, and also many engineering problems concerned with the execution of designs.

The concept of an internal combustion engine was evolved in connection with very early studies in the use of a piston as a means of utilizing heat and pressure. In the year 1680, Christian Huyghens arranged a system of levers and cams operated by a drive shaft emerging from the breech of a small cannon. When the match was applied, a series of muffled explosions shook the room, but the mechanism worked. Then, by means of a series of powder trains, Huyghens began the approach to continuous motion. Obviously, this was neither cheap nor efficient, but it set other men to thinking.

But a whole century was to pass before a machine was accomplishing any work worth mentioning. Robert Street, an Englishman, built a roaring fire under a cylinder containing turpentine, which evaporated and raised the piston. This piston, in turn, drew in air and formed an explosive mixture. A flame was then applied to a touch-hole, the mixture exploded, and the piston was put to work. By frenetically working on the machine, Street managed to get it to pump water.

The tempo of development now accelerated. Philip Lebron, in France, patented a gas engine in 1799 which worked on gas and air fed from separate pumps, detonated, and the force distributed through a valve to the working cylinder. Several names now came into prominence in connection with the internal combustion machine, but one begins to see some semblance to the modern machine of this type in the work of A. V. Newton. He mounted a cast iron plug in the combustion

chamber through which he led a gas flame. The plug was maintained at a red hot temperature so that it would ignite the firing mixture.

In 1862 Beau de Rochas, in France, patented the four-stroke cycle which is the basic principle involved in the present automobile engine. Beau de Rochas was a physicist who never attempted to apply his principle and, in consequence, his name is almost forgotten. However, an attempt is made in the Institute Museum to see that he is not overlooked by visitors to the Prime Movers Section. In 1941 a plaque commemorating the founder of the automobile was presented by the Société des Ingénieurs de l'Automobile and the American Society of Automotive Engineers, and was mounted on a wall in this section.

Nearby, in the same room, will be found one of the Otto gas engines in which the principle was first employed. This dates from the year 1876 and it shows the form of engine that was to supplant all its competitors and became the basis of all the modern gas engines irrespective of their original form of fuel. Light gasoline engines and the heavy oil engines are both variants of this basic type, although they involve many special details.

ERRATUM: In the October 1954 Museum Notes, it was incorrectly stated that William J. Duane was Secretary of State under President Andrew Jackson. This should have read Secretary of the Treasury.

BOOK REVIEWS

ENGINEERING CYBERNETICS, by H. S. Tsien. 289 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $6.50.

Tsien, in this remarkably stimulating book, has made the distinction between engineering cybernetics, which is the engineering *science* dealing with control problems, and servomechanism engineering, which is the complementary engineering *practice*. The book is written for the level of mathematical sophistication possessed by the average research engineer. Demonstrations rather than proofs, and heuristic reasoning rather than rigor, prevail. Thus the average reader is carried along by the drift of Tsien's over-all development, and is not diverted by mathematical side tracks. There is an adequate series of references for the reader who wishes to delve more deeply into the particular sources of Tsien's expository material.

Tsien begins with a discussion of the linear, time invariant sermomechanism. This simple form of control device is considered not from the viewpoint of its particular components, but from the viewpoint of allowing a general theory of design for a specific type of system. Tsien gives the reader a feel for the implication of additional complications in control systems. For example, he demonstrates that in variable coefficient linear systems, as well as in non-linear systems, the usual linear design criterion of stability is inadequate and other criteria arising from the physics of the problem must be used.

The book abounds with concepts which are not to be found in other expository treatments. For example, Tsien discusses non-interacting controls of many variable systems, control design by perturbation theory, optimalization of control, and von Neumann's theory of error control. In a discussion of control design for specified criteria, Tsien illustrates how the control system may be set up analytically as a variational problem, a solution to this problem obtained by a computer, and with this non-linear element, performance superior to that possible with a linear system will be obtained.

It is of interest to note that in the later chapters of this book, Tsien discusses control systems which in their effects approach the type of behavior generated by human operators: optimalization, for example, is a typical human controller action. In optimalized behavior, the operator reads various output and input devices from the machine and then, acting from knowledge and experience, readjusts his controls continuously until the optimum condition is reached. In other words, the properties of the system under control are not encoded in a series of equations, but those properties are continuously sensed.

The concepts of ultrastability and multistability introduced by W. R. Ashley to describe learning by biological organism are presented by Tsien for their application to the design of complex systems. It is almost as though we are reaching the stage in control engineering where, reversing the usual procedure, arguments from the biological system to the mechanical system will become pertinent!

If this enthusiastic reviewer may be permitted a mild cavil, I should like to point out some omissions in Chapter 9 on Linear Systems with Stationary Random Inputs. It seems to me that discussing stationary random inputs insofar as they can experimentally determine the absolute value of the system function is only half the story. The use of cross correlation measurements to get the complex transfer functions (as discussed by Wiesner and Lee and MIT) is the other half of the story and should be mentioned. In addition, a discussion of random inputs without at least a passing reference to the works of Tukey and of Bartlett on estimating the second order properties of such inputs seems incomplete to me.

Aside from the foregoing, it would be difficult to overstate the value of Tsien's book to those interested in the over-all theory of complex control systems. EZRA KRENDEL

STRUCTURE OF MOLECULES AND INTERNAL ROTATION, by San-Ichiro Mizushima. 244 pages, 16 × 24 cm. New York, Academic Press, Inc., 1954. Price, $6.00.

San-Ichiro Mizushima, Professor of Physical Chemistry at the University of Tokyo, has been active in the field of molecular structure for twenty years. Together with his many students and co-workers, he has made the subject of internal rotation in organic molecules his own special hunting preserve. By combining Raman and infrared spectra, dielectric constant, electron and X-ray diffraction, and other measurements, he has shown quite conclusively that rotational isomers exist for molecules such as 1,2-dichloroethane and its relatives. It has been possible to derive values for the energy differences between the isomers and for the heights of the potential energy barriers separating them. These ideas have been extended to more complicated molecules, and have helped to solve problems about the structure of ring-containing compounds and proteins with very long chains.

This excellent book is in a way a summary of that work, or a review of the field in which that work was done. The title is perhaps misleading, since it is not concerned with the structure of all molecules, but only of those showing internal rotation. These include molecules with carbon chain skeletons, polypeptides, and some specialized groups such as alcohols, silanes, etc. Further, the book deals only with the geometrical side of molecular structure. Bond angles and lengths, molecular configurations, vibrational and rotational modes of motion, and other related topics form the substance of this book. It does not deal with the underlying theory. For instance, considerable attention is given to showing that there are three stable but non-equivalent configurations for 1,2-dichloroethane, and to deriving a value for the differences in energy among them. The question as to the causes of these differences is carried no further than to ascribe them principally to steric hindrance. To go further would be beyond the scope of this book.

The book is divided into two Parts. Part 1 contains six chapters on the structure of various types of molecules, including the substituted ethanes and similar molecules, cyclic compounds, alcohols, paraffins, and polypeptides. Part 2 is a kind of appendix, with a chapter on experimental methods and one containing a very detailed discussion of normal mode analysis in these molecules.

This is a thorough and detailed discussion of the geometrical structure of these classes of molecules. It draws on the work of many investigators and the results of many techniques to prove its points. Some of these points are controversial and may be reversed in the future, but this shouldn't detract from its interest and usefulness. It is distinctly not a review or summary for the general reader or student. This reviewer recommends it highly to those working in this field. A. D. FRANKLIN

THE MATHEMATICAL PRACTITIONERS OF TUDOR AND STUART ENGLAND, by Elizabeth G. R. Taylor. 443 pages, plate, 15 × 23 cm. New York, Cambridge University Press, 1954. Price, $9.50.

Most students of the history of a science become familiar with the men of great name in the development of the science, yet few know anything of the long intermediate chain of experimenters who link together the greater men, and who create the climate that makes the influential advances possible. The devoted student should be grateful to the historian who rescues the humbler workers from the shadows. This is precisely what Professor Taylor has done. The names of the illustrious men of the period—Newton, Mercator, Wren, Hooke, Tompion, etc.—do occur in her chapters, but the principal concern is with lesser men, teachers, instrument makers, navigators, land surveyors, and map makers in England during the period between 1485 and 1714, when English science was making its first significant advances.

Before this period, the handling and making of instruments was derogatory to the scholar, unless he happened to be an astronomer or astrologer, who had to keep observational work alive. Parents kept their sons from Oxford, fearing to have them "smutted with the Black Art" of mathematics. However, the intellectual climate changed in the fifteenth century with the revival of learning, the sudden expansion of

the physical horizon, and the introduction of artillery into warfare. England was somewhat slow in getting into the movement but, when she did, a surprising crop of practitioners sprang up on this almost virgin soil.

Professor Taylor's task has been to assemble all the available information on the mathematical teachers, text-book writers, technicians, and craftsmen of the Tudor and Stuart reign, and she has succeeded admirably. Her book is divided into three parts. The first part, called Narrative, will appeal to the general reader, for it is an account of the mathematical practitioners and of the development of ideas, methods, and instruments. This account of a little known epoch in the history of navigation and surveying leaves very little to be desired. The reader does not have to be a student of mathematics to profit from reading it.

The second part consists of brief biographies of nearly six hundred practitioners. This part is the fruit of a truly amazing piece of research and will be an essential tool for all historians of science. It is a priceless biographical summary of early English instrument makers. The third part of the book is a catalogue in chronological order of all the works on mathematical arts and practices the author has been able to locate. The compilation is so comprehensive as to leave no doubt it is complete. The descriptive notes for each entry are in keeping with the general excellence of the work.

The book has been sponsored by the Institute of Navigation, the members of which have reason to congratulate themselves upon their happy choice of an author. The book should encourage others to take up the task of research and the promotion of knowledge in navigation and kindred subjects.

T. COULSON

TRANSISTORS: THEORY AND APPLICATIONS, by Abraham Coblenz and Harry L. Owens. 313 pages, diagrams, illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $6.00.

The transistor was invented in 1948. It has taken six years to reach the consumer market in the circuit of a home radio instrument. Radios operating with transistors completely replacing tubes are now becoming available.

With few exceptions all who were engaged in electron tube applications recognized this achievement almost immediately. As a result, every offering by way of discussion relating to the transistor received overwhelming response. In a brief historical review of the transistor's six-year history the authors, Coblenz and Owens, list a few of the significant semiconductor conferences. Symposia on transistors were arranged by the leading universities, commercial concerns and the professional organizations such as the IRE and AIEE. Against this background of information our book is presented.

Although there is no designation of such a division, it will be observed that the volume is approximately one-third theory, one-third circuit behavior, and one-third manufacturing technique. The authors have stayed completely away from circuits of application. More specialized treatments of the uses of transistors are given in R. F. Shea's "Transistor Circuits" and "Transistor Audio Amplifiers."

Of the three divisions of the book made by the reviewer, theory, circuit behavior and manufacturing, the middle section is the best. In these chapters the transistor is thoroughly analyzed as a circuit component, and the various parameters for grounded-emitter or grounded-collector connections are developed. Here there is an effort to recreate a body of analytic expressions similar to those applying to tubes.

It is doubtful whether much of the information in any part of the book was originally developed by the authors. Those who have followed the technical sessions on transistor theory and practice will agree that a tremendous volume of information has been presented via the lectures and seminars, and that a 300-page book would find some difficulty in treating the three broad divisions mentioned above. One receives the impression in using this book that the authors collected together the lecture notes taken during transistor symposia and filled in the gaps with general information obtained from library reference. This makes for a book of interesting facts on a new subject, but one that is highly confusing. For example, the

theory portion seems to vacillate between levels of education, a general science course for technicians on the one hand and quantum mechanics on the other. The presentation is therefore somewhat "lumpy," overly simplified in places and delving into abstruse subjects in an uncertain manner in other places. A section entitled "Spectra and the Quantum Theory" is devoted exclusively to a description of the prism spectrograph and its importance. In another section Schroedinger's equation is given with almost no explanation. There is a section on blackbody radiation.

The later sections of the book contain even more amazing variety, a sort of "Something to interest everyone" principle. Here the reader will find the "Theoretical Aspects of Nucleon Bombardment" as well as the "Lead Fatigue Test." It would certainly have been better to concentrate on some narrower aspect of the subject of transistors than to strive for an encyclopedic treatment in 300 pages. C. W. HARGENS

ROCKET EXPLORATION OF THE UPPER ATMOSPHERE, edited by R. L. F. Boyd and M. J. Seaton, in consultation with H. S. W. Massey. 376 pages, diagrams, 19 × 25 cm. New York, Interscience Publishers, Inc., 1954. Price, $11.00.

As a result of the acquisition of more than 100 V-2 rockets after the war a powerful tool was available for the exploration of the Upper Atmosphere. Both the United States and Great Britain who shared the spoils used the new investigative medium and it was inevitable that scientists of the two countries should convene around the conference table to present and correlate their findings. This volume is the result of such a conference arranged jointly by the U. S. Upper Atmosphere Rocket Research Panel and the Royal Society's Gassiot Committee.

The book is divided into seven sections of which the first deals with rocket techniques. Dr. Marcus D. O'Day opens this by presenting the problems under investigation. These concern Molecular Dynamics, Ionospheric Investigations and Radiation. The various facilities under contract to the Air Force Cambridge Research Center are given in addition to the plan of investigation. Other significant papers deal with the determination of ambient temperatures, meteor impacts, the use of the V-2 in this study, the Viking as a high altitude sounding rocket and the new "Rockoon" techniques perfected by Prof. J. A. Van Allen and his colleagues.

Section II deals with pressures, temperatures and winds of the upper atmosphere. Results and means of acquiring this information are carefully outlined. In this section comparative results from different approaches are made available.

The composition of the atmosphere is then undertaken and various researchers have investigated the many facets to this problem. Herein are given the chemical analyses of the upper atmosphere and several papers on the ozone content and distribution.

Section IV deals with the ionosphere, solar radiation and geomagnetic variations while the next section is devoted to cosmic ray measurements. Section VI discusses laboratories studies designed to provide possible corroboration with measures made in the upper atmosphere.

Section VII contains some theoretical interpretation of rocket findings and suggested experiments for future rocket soundings.

Until the advent of the rocket most of our knowledge of the upper atmosphere was derived from theoretical considerations and from what little direct evidence which was available. With this new tool important and significant data have been obtained which fill a void in our picture of the earth. *Rocket Exploration of the Upper Atmosphere* presents the first comprehensive picture of our upper atmosphere. It will prove invaluable to the scientist. I. M. LEVITT

SERVOMECHANISM PRACTICE, by William R. Ahrendt. 349 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $7.00.

The author's purpose is to furnish a handbook of information on components and systems of servomechanisms. He distinguishes servomechanisms from other feedback control systems as being those which involve the control of position of an output member, contrasted with systems which control flow, temperature, electrical quantities, etc. The book can hardly be classified as a textbook

but is excellent as a supplementary source of material for the student of advanced control systems. In particular, it is an excellent handbook of reference for information on commercially available components. Chapter titles include Potentiometers, Synchros, Rate Generators, Servo Motors, Friction Dampers, and Rotating Amplifiers, among others. There is included a brief resume of control system theory and an excellent Glossary of Terms which is based on the present status of the work of the AIEE–ASME–ASA joint committee on nomenclature.

Mr. Ahrendt's text may be likened in many respects to the "Components Handbook" by Blackburn, one of the Radiation Laboratory series, which dealt in a similar way with electronic and servo components in general.

BRUCE B. YOUNG

FROM CLASSICAL TO MODERN CHEMISTRY, by A. J. Berry. 251 pages, 14 × 22 cm. New York, Cambridge University Press, 1954. Price, $4.75.

Readers of "Modern Chemistry—Some Sketches of Its Historical Development" will be glad that its author has returned to the subject in this volume. He avails himself of the opportunity to treat with some topics that were omitted or only inadequately touched upon in the earlier work. Instead of following a chronological order in the developments, Dr. Berry has sensibly divided his material into subjects and treats them separately. After an interesting chapter on the theory of chemistry the studies are: Vicissitudes of Heat; Aspects of Classical Electricity and Electrolysis; Physical Optics and Chemistry; Molecular Magnitudes; Analytical Chemistry; Chemical Formulae; Valency, Radicals and Constitutions; and Kinetic Energy.

It will be seen that the author has appreciated the corresponding history of physics and its relation to chemistry. In connection with this, one cannot refrain from mentioning the praiseworthy attention given to some workers who do not always get their full meed of praise in lengthier studies of their sciences, as, for instance, Cavendish in electricity, and Young in the extension of Newton's work on the color of thin plates and films.

In a book of this size the subjects necessarily are treated in what the author calls a "somewhat impressionistic" method, but the chapters form satisfactory introductions to the various subjects for those who have an elementary knowledge of chemistry. (We do not recommend the book for those who have no knowledge of the subject.) The more serious student may benefit from using the bibliographies appended to each chapter. The references in these are to original works, rather than to other histories on the subjects.

Dr. Berry's treatment of his subject is concise but he has contrived to compress a great deal of information into the space at his disposal. In short studies of this nature it is difficult to maintain a balanced perspective, yet the author does this in scholarly fashion without apparent effort.

T. COULSON

INTRODUCTION TO THEORETICAL MECHANICS, by Robert A. Becker. 420 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $8.00.

This book is designed to cover the range for an introductory course in classical mechanics in a curriculum for Engineering Physics. As such, it represents a useful contribution. It covers a well chosen range of topics and problems. Though the book offers very little of an original contribution to the subject, its general exposition is good. The selection of problems, however, follows closely that found in more orthodox texts.

On the emphasis placed on utilizing problems for a better understanding of physical principles, some criticism may be in order. On p. 68, the author discusses the stresses at section D of a beam. The nature of the interaction between AD and DB should be clearly noted—the author gives only one aspect of the stress. Moreover, equations 3–37 and 3–38 are obtained from the conditions of equilibrium of the portion DB, noting the mutual action between AD and DB, and not from the reasoning in Sec. 3–14. Again, in the problem of the bead on a rotating ring, an alternate procedure is to regard the mass m as actually having two degrees of freedom. The reaction of the ring on the bead then appears as a force acting on the coordinate ϕ from which a more general

expression is obtained than that given by 13–59. In fact, it is of interest from a dynamical aspect, to show that the Lagrangian equation of the ϕ coordinate is actually the equation for the changes in angular momentum for the isolated system containing only the bead. Assuming $\dot{\phi} = \omega =$ constant is then a special case, wherein only the centrifugal and Coriolos components appear in the reaction. The method of varying the constraint can best be used for determining a reaction when the constraint does no work in the actual system. The reaction R of the constraint using ϕ as a coordinate, does work on the system even though $\dot{\phi} = \omega =$ constant. It provides the source of power, $Rb\ \omega \sin \theta$, for the required rate of change of energy, to maintain $\dot{\phi} = \omega =$ constant.

On pages 26–27, Newton's third law is discussed, Figs. *a*, *b*, and *c*, wherein an *external* force is applied to the system. On this assumption the contact reaction of the Earth R is not equal to $\bar{W} + \bar{F}$. Moreover, the inertial system for the block is not the acceleration relative to the Earth, when considering the force F acting as an external force to block plus Earth. The reaction $-R$ (which should be shown on the Earth Fig. *c*) with $-W$ is an unbalanced force causing acceleration of the Earth. The block must sustain the same acceleration downward, with $\bar{W} + \bar{F}$ and $\bar{R}$ acting on the block, as an unbalanced force. The acceleration, though small, cannot be neglected, and is not the acceleration relative to the Earth.

The author's discussion of accelerated reference systems is good. This is followed by the motion of rigid bodies in three dimensions, wherein the rather difficult subject of moments of inertia and products of inertia, principal axis and ellipsoid of inertia is presented with clarity. The author then extends the rigid body motion to gyroscopic problems as the equations for the motion of a top. Euler's coordinates are introduced. The extension to gyroscopic systems might give more emphasis to problems illustrating physical concepts in space-time changes in angular momentum beyond the more formal application of Euler's equations.

The introduction of generalized coordinates with the Lagrangian and Hamiltonian equations follows closely the presentation given by other texts on the subject. In article 13–4 for Lagrange's equation for a system of particles, it is not permissible to omit the forces of constraint for the equations of motion for the ith particle in equation 13–61. The $\bar{X}_i$, $\bar{Y}_i$, $\bar{Z}_i$ include all the forces acting on m_i. In the form 13–62, however, the right side can omit the forces of constraint, since its derivation ultimately depends upon the equation of virtual work, in which the forces of constraints do no work.

The final chapters deal with vibrating systems and normal coordinates, followed by a concluding chapter on Vibrating Strings and Wave Motion. In the discussion of normal coordinates, it is important to point out that damping forces in general cause couplings between these coordinates, which is inhibited in the particular problems chosen in the text.

While this book covers the usual range in so-called classical mechanics, it seems that further opportunities are open to either extending classical mechanics to actual problems in engineering and applied physics, or, on the other hand, to cover a transition range from classical to quantum mechanics.

RUPEN EKSERGIAN

WELDING FOR ENGINEERS, by Harry Udin, Edward R. Funk and John Wulff. 430 pages, diagrams, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1954. Price, $7.50.

As a means of fabrication, welding is more and more supplanting riveting. It is cheaper, quicker and easier. However, it presents many problems that riveting does not. As a matter of fact, in order to thoroughly understand all the ramifications of welding, one must be something of a mechanical engineer, metallurgist, chemist and electrical engineer. One must know something of structural design, the basic elements of metallurgy, the chemistry of flames in gas welding, and something of electricity and physics for arc and resistance welding. All these aspects of welding are discussed in *Welding for Engineers*.

Written by Harry Udin and John Wulff of the Department of Metallurgy of M.I.T. and Edward R. Funk, Engineering Consultant to the Goodyear Aircraft Corporation,

Welding for Engineers offers a basic introduction into the theory and practice of welding. Following an introduction where the broad field of welding is discussed, cold welding, hot pressure welding and resistance (butt, spot, seam and projection) welding are described. The metallurgical reactions encountered in resistance welding, and the heat and temperature developed during fusion welding, are explained. The arc-welding process with both the permanent and consumable electrode is next discussed. This includes the nature of the arc, the electric circuit and the various types of electrodes. Sources of chemical heat for welding, such as oxyacetylene and oxyhydrogen, and oxygen for cutting are discussed. The transformations that occur in steel during the heating and cooling cycles of fusion welding are explained.

Not all steels are weldable, or as easily welded as others. This fact is an important consideration in structural design. While a steel may possess certain desirable physical properties, if it can't be easily welded it may be rejected. These facts and others including the proper choice of electrode are explained under the weldability of steels. Following two chapters on braze welding, the book is concluded with the mechanical effects of welding, that is, distortion and residual stresses with ways to combat the former and measure the latter, and principles of weld inspection and testing.

The book is simply written, elementary, in that it starts from the basic facts of the various subjects of welding, and easily understood. For one who wants to know what happens in welding, *Welding for Engineers* is highly recommended. E. W. HAMMER

BOOK NOTES

A BUDGET OF PARADOXES, by Augustus DeMorgan. Two volumes bound as one. Reissued, unabridged, with a new introduction by Ernest Nagel. 387 pages, 14 × 21 cm. New York, Dover Publications, Inc., 1954. Price, $4.95.

Scientists in all fields will be interested in this reprinting of a 19th Century classic, and in the introduction by Ernest Nagel, who states: "Surely there are many attentive readers who will not be able to repress the comment in connection with some of these figures: 'There but for the grace of God go I.'" For witty commentary on these paradoxers who challenged generally accepted scientific beliefs with their own unsubstantiated theories, DeMorgan cannot be surpassed. Here are set forth the views of the fanatics, objectors, romantics, etc., who undertook to prove wrong such things as Newtonian physics—with caustic remarks by the author. If you are thinking of proposing a way to square the circle, initiate perptual motion, upset the laws of gravity—it is suggested that you first read the *Budget*. It will open your eyes to the ways of paradoxers, and save you considerable embarrassment later on.

AMERICAN MEN OF SCIENCE. VOLUME I, PHYSICAL SCIENCES, edited by Jaques Cattell. Ninth edition, 2180 pages, 21 × 29 cm. New York, R. R. Bowker Co., 1955. Price, $20.00.

The greatly expanded list of men who deserve a place in *American Men of Science* has forced the publication of this ninth edition in three volumes. Volume I, just published, lists over 44,000 living Americans and Canadians who have contributed to the physical sciences, including physics, mathematics, chemistry, biological chemistry, meteorology, oceanography, optics, acoustics and research engineering. Authoritative information is given on each person's position, education, professional background, special fields of research, membership in professional societies, date of birth, marital status and number of children. Volume II on Biological Sciences will appear in the fall of 1955; Volume III on Social Sciences is scheduled for the spring of 1956. In cases where a person's field of interest lies both in the Physical Sciences and in Biological Sciences (as in Biochemistry), his biography is listed in only one volume, with a cross-reference in the other, so that his name appears in both Volumes I and II. The new edition will be welcomed in scientific circles, for it performs an unique service to science.

PACKAGING ENGINEERING, by Louis C. Barail. 407 pages, diagrams, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1954. Price, $9.50.

This new book will no doubt become the standard reference work in the packaging industry. It covers types of containers, machinery for manufacture of containers, packaging design, protection against deterioration, labeling, testing and exporting. It is well illustrated and covers all packaging materials —even the very recent ones are described in a special chapter at the end of the book. The chapter on packaging requirements of the Armed Forces should be especially valuable to industries that sell to the Government. The chapter on packaging for export is enhanced with a glossary in French and Spanish, and includes pages of valuable information on gross and net weights of all sorts of exports, with packaging requirements for each. The book should be of great value not only to the executives and supervisors in the packaging industry itself, but to such allied enterprises as airlines, railroads and other carriers, and to all industries which package their products.

ORGANIC CHEMISTRY, by Lewis F. Hatch. 324 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $4.50.

Professor Hatch has written this text for a one semester (or less) course for students in many fields, but who must have an understanding of organic chemistry. After presenting, briefly, the standard concepts of organic chemistry, the author covers such special fields as biological chemistry, agricultural chemistry, medicinals, pharmaceuticals, petroleum, petrochemicals, plastics and synthetic fibers. Practical aspects of each subject are given, as well as theoretical ones.

ORGANIC SYNTHESES, VOLUME 34, edited by William S. Johnson and others. 121 pages, diagrams, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1954. Price, $3.50.

Volume 34 of this annual series covers 34 compounds, giving satisfactory methods for their preparation. The subject index includes references for Volumes 30 through 34, and has been carefully prepared to be the most use to chemists. Organic chemists have come to depend on the reliability of these preparations, and will want to add the present volume to their library.

CATALOGUE OF THE H. H. ROGERS COLLECTION OF SHIP MODELS. 117 pages, illustrations, 17 × 25 cm. Annapolis, United States Naval Institute, 1954. Price: $2.00 (cloth); $1.50 (paper).

This beautifully printed catalogue illustrating the Henry Huddleston Rogers collection of ship models, should induce those interested in ships to go to the U. S. Naval Academy in Annapolis to see the originals. The catalogue describes, by number, all of the models, giving dimensions, materials, etc., as well as various photographic views of each model. There are 108 models, some of which are valued as high as $17,000. If you aren't able to visit the Academy to see these fascinating ship models, the catalogue will serve as a "second-best," for the photographs are exquisitely done.

PUBLICATIONS RECEIVED

ASPECTS OF DEEP SEA BIOLOGY, by N. B. Marshall. 380 pages, plates, illustrations, 16 × 24 cm. New York, Philosophical Library, 1954. Price, $10.00.

METHODS OF MATHEMATICAL PHYSICS. VOLUME I, by R. Courant and D. Hilbert. First English edition, 561 pages, 16 × 24 cm. New York, Interscience Publishers, 1953. Price, $9.50.

ABBOTT'S NATIONAL ELECTRICAL CODE HANDBOOK, revised by Charles L. Smith. Eighth edition, 642 pages, diagrams, illustrations, 15 × 21 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $7.50.

CONVEYORS AND RELATED EQUIPMENT, by Wilbur G. Hudson. Third edition, 524 pages, diagrams, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1954. Price, $9.00.

MAGNETIC CONTROL OF INDUSTRIAL MOTORS, by Gerhart W. Heumann. Second edition, 714 pages, illustrations, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1954. Price, $9.50.

APPLIED GEOPHYSICS, by A. S. Eve and D. A. Keys. Fourth edition, 382 pages, illustrations, 14 × 22 cm. New York, Cambridge University Press, 1954. Price, $7.50.

CHEMICAL CALCULATIONS, by H. V. Anderson. Sixth edition, 305 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $4.75.

ACTIONS OF RADIATIONS ON LIVING CELLS, by D. E. Lea. Second edition, 416 pages, plates, 15 × 22 cm. New York, Cambridge University Press, 1955. Price, $6.00.

FLUORIDATION AS A PUBLIC HEALTH MEASURE, edited by James H. Shaw. 232 pages, illustrations, 16 × 24 cm. Washington, American Association for the Advancement of Science, 1954. Price: $4.00 (members); $4.50 (non-members).

BASIC TELEVISION, by Bernard Grob. Second edition, 660 pages, diagrams, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $6.00.

ALGEBRA FOR COLLEGE STUDENTS, by William M. Whyburn and Paul H. Daus. 290 pages, 15 × 22 cm. New York, Prentice Hall, Inc., 1955. Price, $4.25.

STORAGE BATTERIES, by George Wood Vinal. Fourth edition, 446 pages, illustrations, 16 × 23 cm. New York, John Wiley & Sons, Inc., 1955. Price, $10.00.

AGRICULTURAL PROCESS ENGINEERING, by S. M. Henderson and R. L. Perry. 402 pages, charts, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1955. Price, $8.50.

THE CHEMISTRY OF PORTLAND CEMENT, by R. H. Bogue. Second edition, 793 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1955. Price, $16.50.

PETROLOGY FOR STUDENTS, by A. Harker. Eighth edition, 283 pages, illustrations, 13 × 19 cm. New York, Cambridge University Press, 1955. Price, $3.50.

SIMPLIFIED DESIGN OF STRUCTURAL STEEL, by Harry Parker. Second edition, 244 pages, diagrams, 14 × 20 cm. New York, John Wiley & Sons, Inc., 1955. Price, $5.75.

PRESTRESSED CONCRETE, by Gustave Magnel. Third edition, 345 pages, diagrams, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $8.00.

HUGH ROY CULLEN, by Ed Kilman and Theon Wright. 376 pages, drawings, 15 × 23 cm. New York, Prentice-Hall, Inc., 1954. $4.00.

SOIL, by G. V. Jacks. 221 pages, 14 × 21 cm. New York, Philosophical Library, 1954. Price, $5.00.

ADVANCED MATHEMATICS FOR ENGINEERS, by H. W. Reddick and F. H. Miller. Third edition, revised by F. H. Miller, 548 pages, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1955. Price, $6.50.

JET. THE STORY OF A PIONEER, by Sir Frank Whittle. 320 pages, plates, 15 × 22 cm. New York, Philosophical Library, 1954. Price, $6.00.

APPLIED X-RAYS, by George L. Clark. Fourth edition, 843 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $12.50.

The following list of titles are publications received from the National Bureau of Standards. All are published by and are available from the Government Printing Office, Washington 25, D. C. Foreign remittances must be in the U. S. exchange and should include an additional one-third of the publication price to cover mailing costs.

REPORT OF THE THIRTY-NINTH NATIONAL CONFERENCE ON WEIGHTS AND MEASURES. NBS Miscellaneous Publication 212, 108 pages, 15 × 23 cm. 1954. Price, $0.50.

TABLE OF SINE AND COSINE INTEGRALS FOR ARGUMENTS FROM 10 TO 100. NBS Applied Mathematics Series 32 (reissue of Math. Tables 13), 186 pages, 21 × 27 cm. 1954. Price, $2.25.

RADIOACTIVE-WASTE DISPOSAL IN THE OCEAN. NBS Handbook 58, 31 pages, 13 × 20 cm. 1954. Price, $0.20.

THERMAL CONDUCTIVITY OF METALS AND ALLOYS AT LOW TEMPERATURES, by Robert L. Powell and William A. Blanpied. NBS Circular 556, 68 pages, 20 × 26 cm. 1954. Price, $0.50.

PRECISION LABORATORY STANDARDS OF MASS AND LABORATORY WEIGHTS, by T. W. Lashof and L. B. Macurdy. NBS Circular 547, 24 pages, 14 tables, 20 × 26 cm. 1954. Price, $0.25.

PERMISSIBLE DOSE FROM EXTERNAL SOURCES OF IONIZING RADIATION. NBS Handbook 59, 79 pages, 13 × 20 cm. 1954. Price, $0.30.

PROTECTION AGAINST RADIATIONS FROM RADIUM, COBALT-60, AND CESIUM-137. NBS Handbook 54, 60 pages, 18 figs., 10 tables, 13 × 20 cm. 1954. Price, $0.25.

FORMULAS FOR COMPUTING CAPACITANCE AND INDUCTANCE, by Chester Snow. NBS Circular 544, 69 pages, 37 figs., 20 × 26 cm. 1954. Price, $0.40.

PHOTOGRAPHIC DOSIMETRY OF X- AND GAMMA RAYS, by Margarete Ehrlich. NBS Handbook 57, 28 pages, 12 figs., 5 tables, 13 × 20 cm. 1955. $0.15.

CHEYENNE MOUNTAIN TROPOSPHERIC PROPAGATION EXPERIMENTS, by A. P. Barsis, J. W. Herbstreit and K. O. Hornberg. NBS Circular 554, 39 pages, 46 figs., 3 tables, 20 × 26 cm. 1955. Price, $0.30.

MINIATURE INTERMEDIATE-FREQUENCY AMPLIFIERS, by Robert K-F Scal. NBS Circular 548, 46 pages, 36 figs., 20 × 26 cm. 1954. Price, $0.40.

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS. NBS Applied Mathematics Series 34, 105 pages, 21 × 27 cm. 1954. Price, $2.00.

CURRENT TOPICS

New Medium for Telephone and Television Transmission.—A new and radically different medium for transmitting television and telephone conversations over long distances has been used successfully in experiments, Bell Telephone Laboratories announced recently.

The new medium, a long distance waveguide, is markedly different from modern cable or radio relay systems—it uses hollow metallic tubes roughly two inches in diameter.

Bell scientists believe the new waveguide may someday simultaneously carry tens of thousands of cross-country telephone conversations along with hundreds of television programs. Top capacity for the most modern of coaxial cable systems is 1860 two-way telephone conversations or 600 such telephone conversations and two TV programs simultaneously on a pair of coaxial tubes. Modern coaxial cables have eight such tubes, two of which are kept as spares for emergencies.

Waveguides made of solid metal tubing—roughly like a metal water pipe—have been widely used for some time for short distances. It would be possible to use these solid metal tubes for long distances if they were perfectly straight, but this is impractical.

The new long distance waveguide is also a hollow tube, but it is constructed of a thin copper wire, very tightly coiled—like a spring under pressure—and wrapped inside a flexible outer coating which holds the coiled wire in place. This type need not be straight and can actually carry signals around corners.

Experiments indicate that both the solid tube type waveguide and the new coiled wire or "helical" type of waveguide can be used together in communications systems, the first for short distances and the latter for long distances.

Recent experiments at Bell's Holmdel, N. J., laboratory indicate that the new waveguide transmission is extremely promising and might, when fully developed, join coaxial cable and radio relay as an everyday transmission medium for the Bell Telephone System. Engineers hope to use the new waveguide in a variety of ways; one of these might be with a heavy protective coating so that the hollow tubes are run underground.

Although this new form of transmission is still in the experimental stage, a recent long distance test was made at Holmdel in a copper pipe 500 ft. long. Engineers bounced signals back and forth in the tube for distances of 40 miles. They calculated that in comparison, the same waves could have traveled only 12 miles in a coaxial cable with the same loss in strength.

The new transmission system operates in a frequency range so high that it has never before been put to practical use for communications. As a matter of fact, this range is so high that engineers do not have a name for it yet although the short waves used are known as millimeter waves. The "super high" frequency range established by the Federal Communications Commission goes up to 30,000 megacycles. The carrier frequency for the new waveguide is about 50,000 megacycles.

A major difference between transmission through the new waveguide and through previous systems is that the higher the frequency in the waveguide, the less the loss through attenuation. This is exactly the reverse of other forms of transmission.

Beyond the prospect of an improved transmission with the long distance waveguide is the possibility of learning how to use tiny wave lengths on the order of one millimeter, about a 25th of an inch. If this should become possible, the waveguide of the future may be no thicker than a fountain pen and still carry tens of thousands of telephone messages.

***USS Nautilus* an Atomic Age Engineering Triumph.**—The atomic powered *Nautilus,* now undergoing sea tests, has been described by its chief Naval developer as "ninety percent engineering and only ten percent physics." That description was given by Rear Admiral H. G. Rickover in a tape recorded radio program for broadcasting during National Engineers' Week, February 20–26. On the recorded program, Admiral Rickover, who guided the atomic submarine project from a drawing board idea to ocean-going reality, cited the building of the *Nautilus* as an outstanding engineering achievement of the atomic age.

Pointing out that the famous equation of Einstein which indicates the relationship between matter and energy has been known for years, Admiral Rickover stated that "what we are seeing now is a concentrated engineering effort directed toward making practical use of that and other scientific theories."

"The peace time progress that will be made in moving the atom into industry will be a progress based on engineering success," he said.

The Admiral emphasized that the atomic submarine project had resulted in the "accumulation of a vast amount of technical know-how concerning the design and construction of small scale atomic reactors."

He added that this technical knowledge should prove invaluable to engineers for the industrial development of central station atomic power plants.

New Atomic Clock.—Development of an atomic "clock," the Cesium Atomic Frequency Standard, which is fantastically accurate, has been announced by Dr. Jerrold R. Zacharias, director of the Laboratory for Nuclear Science at Massachusetts Institute of Technology. He is working on a model so precise that if it had been ticking away since the time of Christ it would now be only one-half second "wrong." Manufacture of a commercial model of the clock has been started by a company in the Boston area and it will be available for many uses within a year.

The primary purpose in developing the apparatus, according to the physicist, was to build a tool for scientific research. He expects to employ it, for example, in a test of Einstein's General Theory of Relativity, by which it is supposed that gravity would cause two clocks, one on top of a mountain and one at the foot, to vary in time-keeping. He hopes to undertake such a test next year, he said, perhaps on the Jungfrau in Switzerland, where the mountaintop is easily accessible and can be seen from the foot.

Other possible uses for such a precise timing device would include:

Navigation. Present methods enable a ship to locate its position within only five or ten miles. The cesium device will give a very precise location.

Astronomy. Very minute changes in the movement of the earth, moon, stars and other bodies can now be measured.

Geophysics. The degree of rigidity of the earth, which is not as solid as laymen might suppose, can be determined.

Communication. Synchronization of events in different places can be attained with remarkable accuracy.

Time-keeping in the device is controlled by the oscillation of electrons in the cesium atom, just as a pendulum controls it in an old-fashioned clock or a balance wheel controls it in a watch. As an electron revolves around the nucleus of an atom, it "wobbles" very slightly but at a constant rate. This unvarying rate of oscillation is reflected in the frequency of waves that are emitted.

Cesium is an element, one of an atomic group that includes sodium and potassium, which has a frequency of approximately 9192.632 megacycles per second, and it is this frequency that serves as the unit of time in the atomic clock.

One one-hundredth of a gram of cesium, about the size of a pinhead, is enclosed in a metal crucible. Under heat, about that of boiling water, the cesium shoots a stream of atoms through a hole in the crucible. The atoms strike a detector screen and their frequency is reported through complex apparatus.

The cesium emits atoms at a rate of about one million per second, but the loss is only about one-millionth of a gram per day and, for all practical purposes, the clock is perpetual.

While for scientific work, clock hands would not provide sufficiently precise timing, the device can be connected with a conventional but finely calibrated clock face for use in such operations as navigation. An experimental model has been constructed at M.I.T. Commercial models will be contained in a cabinet about the size of two grandfather's clocks.

Standard time, with one second equal to 1/86,400th of the mean solar day and generally measured by the frequency of the oscillations of a crystal, is accurate to one part in ten to the ninth power. The Atomic Frequency Standard is accurate to one part in ten to the tenth power, or 10,000,000,000. Dr. Zacharias expects, through further development, to obtain accuracy of one part in ten to the twelfth power and says, "This is just the beginning."

Associated with Dr. Zacharias in the work have been James G. Yates, an English electrical engineer who was a visiting professor at M.I.T. and is now at Trinity College, and Robert D. Haun, a research assistant in the Department of Physics at M.I.T.

The device was developed in the Research Laboratory of Electronics with support in part from the Signal Corps, the Office of Scientific Research, the Air Research and Development Command and the Office of Naval Research.

Dr. Zacharias said, "Development of the Cesium Atom Frequency Standard is a first-rate example of the conversion of post-war basic science into technology. And it is an example of how the combination of theoretical work, experimental work and industrial production can yield something of use in many fields which will make further research possible."

Tubular Heater for Diesel Engines.—A new tubular heater for diesel engines—designed for quick-starting in sub-zero weather—has been developed by the General Electric Company's Industrial Heating Department. Product of several years of research at the department's Pittsfield, Mass., plant, the tubular heater enables an operator to start a diesel engine under all weather conditions, according to G-E engineers.

A component part of the diesel engine, the tubular heater is about the

size of an ordinary lead pencil and is installed into the engine's precombustion chamber. When the engine is cranked, power from the batteries is transmitted to the heater which, in 30 seconds, reaches a temperature of 1800 F., the engineers said. This extreme heat materially increases the temperature in the precombustion chamber, thus assisting in igniting the fuel as it is injected into the chamber.

Tests conducted over several years by the Caterpillar Tractor Company have established the tubular heater as a reliable electrical starting system. According to their engineers, a novice operator easily started a D-4 tractor, equipped with the G-E device, which had been parked outdoors for an entire winter. In still another realistic performance test, the same tractor was started within an average cranking time of 1.15 seconds following a weekend period in which it had been subjected to sub-zero temperatures with the engine housing cover removed.

Records maintained by Caterpillar engineers during their test period show that pre-heat time of the tubular heater varied from zero to four minutes depending on the temperature factor.

Methods to Decontaminate Radioactive Water.—The possibility of radioactive contamination of public water supplies from the use of nuclear weapons or from improper disposal of waste from atomic reactor installations, research organizations or hospitals has emphasized investigations of methods of dealing with such contamination.

Removal of radioactive contaminants from water by ion exchange slurry is among the latest of these methods tested by the Sanitary Engineering Branch of the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Virginia.

The method consists in removing the radioactive contaminants from the water by the addition of commercially available ion exchange resins. Attracted to the resins, the radioactive ions settle out with them after the solution has been agitated.

In the laboratory jar tests conducted by ERDL, the ion exchange resins were added to tap water contaminated with radioactive materials to a level approximating that expected in a water supply after the detonation of a nuclear type bomb.

Tests indicated that under certain conditions the ion exchange batch slurry treatment decontaminates radioactively contaminated water to a level suitable for emergency drinking purposes within 30 minutes. Because equilibrium is reached rapidly in the batch treatment it is faster, although sometimes less efficient, than the column type operation in which ion exchange materials are also used.

Tiny Tantalum 'Lytics for Large Values of Capacitance.—A new miniature "cup" electrolytic capacitor, the fifth design to be added to the complete Sprague "Tantalex" line, has been announced by the Sprague Electric Company. Designated Type 104D, these new capacitors are ideal for low voltage applications where they provide values of capacitance up to 30 MF in units less than $\frac{1}{10}$ cu. in. in volume. The use of tantalum, the most stable of all anodic film-forming materials, further gives them unusual stability of performance. They exhibit no shelf aging under long periods of test, and have extremely low leakage current.

Normal operating temperature range of the new Type 104D Tantalex Capacitor is from -55 to $+85$ C. at rated working VDC. Units may be obtained on special order which operate at 100 C. with a voltage derating of 15 per cent.

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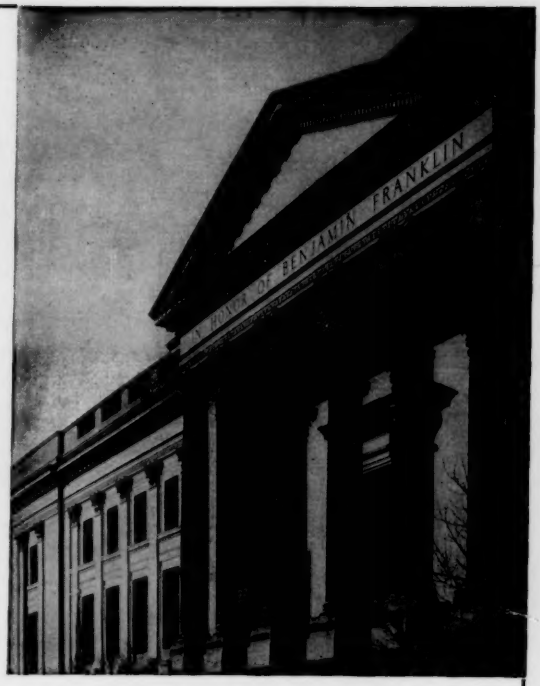

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